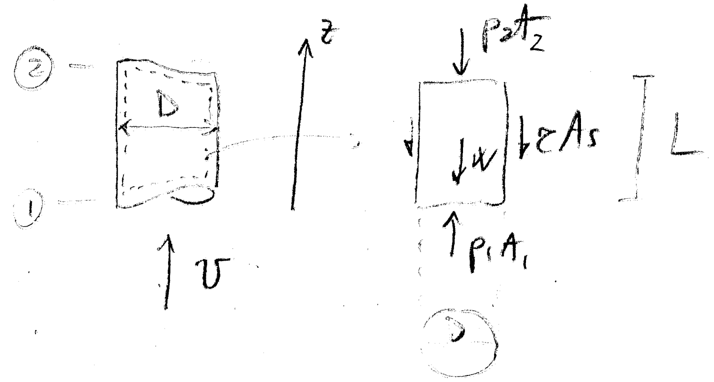
13A



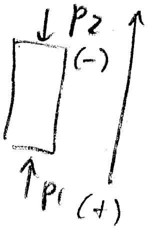
1) τ = shear stress acting on A_s

$A_s = \pi D \cdot L$

2) pressure acting on the 2 surfaces normal to the flow

$A_1 = A_2 = \frac{\pi D^2}{4}$

P_1, P_2



C.S = A_1, A_2, A_s

Let us write the Mom. eq. along z.

3) weight γV_{OL}

$$\sum F_z = \underbrace{(P_1 - P_2)\frac{\pi}{4}D^2 - \tau \pi D L}_{\text{surface forces}} - \underbrace{\rho g \frac{\pi D^2}{4} \cdot L}_{\text{body surface}}$$

surface forces ACTING THROUGH THE C.S.
↓
need some form of interaction through the surface (contact, friction, pressure)

body surface ACTING ON THE MASS OF FLUID in the C.V.

The term

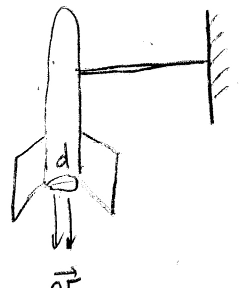
$\frac{d}{dt}\int_{C.V} \bar{v}\, \rho \, dV_{ol}$ is the accumulation rate of MOMENTUM in the C.V.

If the flow is steady $\frac{d}{dt}\int \ldots = 0$
and the forces "
and the C.S. is steady

BB

Example FLUID Jets — Thrust of a rocket.

Ⓐ Given :



m rocket = 40 g

d exhaust = 1 cm

$v =$ 450 m/s

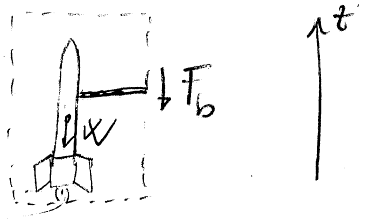
ρ = 0.5 Kg/m^3

Ⓑ Assume no Mom. change in the rocket
Assume $P_{exhaust} = P_{AMBIENT}$ → free jet.

Ⓒ Find ?

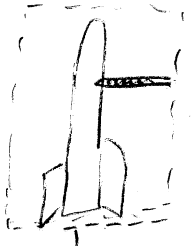
F_b on the beam supporting the rocket

1) choose c.v.



Force DIAGRAM

F_b, W (No press.)



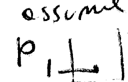
$\downarrow \dot{m} v_{OUT}$

Momentum DIAGRAM

ASSUME UNIFORM CROSS sect.

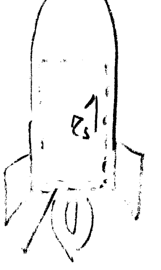
$$\sum \dot{m}_{OUT} v_{OUT} = \dot{m} \cdot (-v) = \rho A v(-v) = -\rho A v^2$$

important. since we assume



$P_1 = P_{AMB.}$ then $P_1 - P_{AM} \cdot A = 0$

Why don't we choose this c.v ?



more complicate

1) how the fluid is moved

2) friction

Mom eq. in z.

$$\sum F_z = \cancel{\frac{d}{dt}\int_{CV} v_z \rho \, dV_{ol}}^{=0 \text{ by ASSUMPT.}} + \sum_{C.S.} \dot{m} v_{OUT\,z} - \cancel{\sum_{C.S} \dot{m} V_{IN\,z}}^{=0}$$

$$-F_b - W = \underbrace{-F_b - mg = -\rho A v^2}$$

$$F_b = \rho A v^2 - mg$$

Question why $F_b \downarrow$? → without F_b the rocket would move up, so F_b is pushing the rocket DOWN.

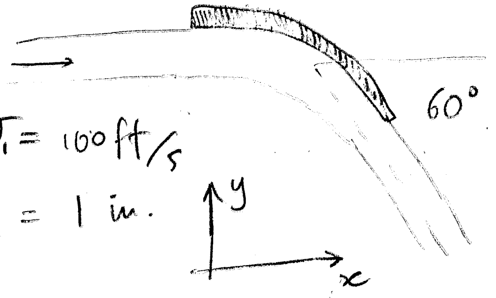
13 D Water deflected by a vane

ASSUME → neglect gravity
steadiness

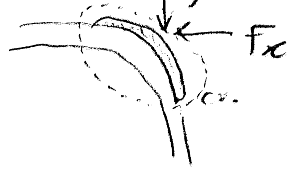
$\boxed{?\text{) Force exerted on the vane by the jet.}}$

$V_1 = 100 \text{ ft/s}$

$d = 1 \text{ in.}$



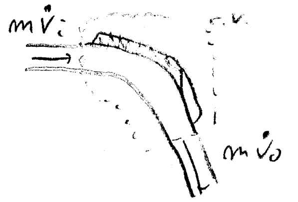
Force Diag. — C.V. include the VANE! so we do not worry about the shear stresses



$$\sum \vec{F} = \cancel{\frac{d}{dt}\int_{CV} \rho \vec{V}\, dV_{ol}}^{=0 \text{ steady}} + \sum_{CS} \dot{m}_o \vec{V_o} - \sum_{CS} \dot{m}_i \vec{V_i}$$

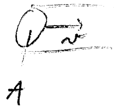
$$F = F_x \hat{x} + F_y \cdot \hat{y}$$

Mom. Diag.



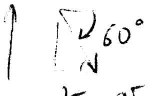
CONTINUITY $= \dot{m}_i = \dot{m}_o = \dot{m} = \rho A v$

as $A = \text{const}$, $\rho = \text{const} \rightarrow v = \text{const}$.



$$\sum_{in} \dot{m} V_i = \dot{m} \cdot V_i \,\hat{x}$$

$$\sum_{OUT} \dot{m} V_{OUT} = \dot{m}\, v \cos\alpha \,\hat{x} - \underset{\text{opposite to } y}{\dot{m}\, v \sin\alpha}\, \hat{y}$$



$V_x = V \cos\alpha$

$$\text{along } \hat{x} \rightarrow F_x = \underbrace{- \dot{m} V_i}_{-\sum \dot{m} V_{in}} + \underbrace{\dot{m} V_o \cos\alpha}_{\sum \dot{m} V_{OUT}}$$

$$\text{along } \hat{y} \rightarrow -F_y = - \dot{m} V_o \sin\alpha$$

$\boxed{V_i = V_o = V}$

$$F_y = \dot{m} v \sin\alpha = \rho A v^2 \sin\alpha = 91.8 \text{ lbf}$$

$$F_x = \dot{m} v (1 - \cos\alpha) = 53 \text{ lbf}$$

the force of the jet is opposite!

114a

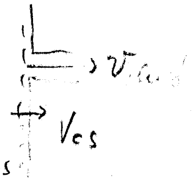
Moving control volumes.

So far: stationary fixed CONTROL VOLUME: Remember CLASS ROOM

Rem: $$\sum F_{ext} = \frac{d}{dt}\int_{C.V} \bar{v}\,\rho\, dV + \int_{CS} \bar{v}\,\rho\,\vec{V}\cdot d\vec{A}$$

1) velocity of the fluid relative to the control surface → indicates MOM IN and OUT

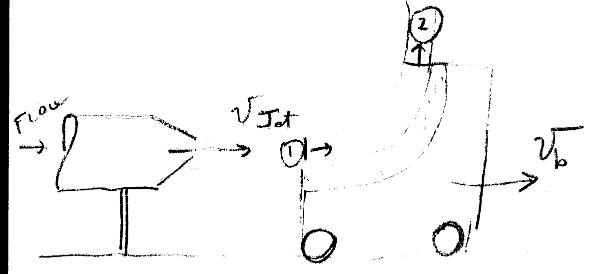
So if the control surface moves, what still matters is the relative velocity!



Fluid moves out if $v_{fluid} - V_{c}$

2) However $\bar{v}$ is relative to an inertial reference frame → NO ACC. NO ROTAT. BUT could move.

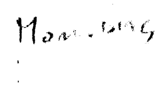
Example ① C.V. MOVING CONSTANT SPEED



Jet impinging on a BLOCK, MOVING AT CONST vel. v_b

what is F friction on the block!?

FD



Mom. DIAG

$$\sum F_x = \frac{d}{dt}\int_{CV} v_x\,\rho\, dV_{\alpha} + \sum_{CS} \dot{m} v_{ox} - \sum_{CS} \dot{m} v_{ix}$$

Mom eq.

$x \rightarrow \quad \sum F_x = -F$

|| if ① LAGRANG. REF FRAME MOVING WITH THE CART AT SPEED U_B Ref. frame → CART moving at const velocity $\quad \frac{d}{dt}\int_{CV} \dots = 0$

2) $\dot{m}V_{ox} = 0$ / NOTHING ALONG X

1) $\dot{m}V_{ix} = \underbrace{\dot{m}(v_J - v_b)}$; $\dot{m} = \int \rho V dA = \rho A(v_J - v_b)$

$-F = \underbrace{\rho(v_J - v_b)A}_{\dot{m}} \cdot (v_J - v_b)$

$-F = -\dot{m}(v_J - v_b) =$

$-F = -\rho A(v_J - v_b)^2$

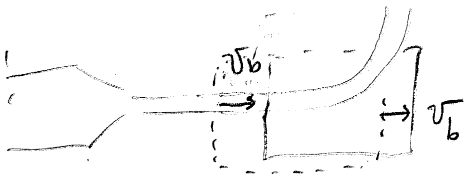
14 b) ② EUL. REF. FRAME

Let us now consider
the ref. frame on the nozzle → fixed

$$\frac{d}{dt}\int_{CV} \dots = 0$$ still ... NO MOM. ACCUMULATION

$$\underline{\underline{\sum_{CS} \dot m V_{ix} = \dot m V_J}}$$ MOM. IN THE CV

$$\underline{\underline{\sum \dot m V_{ox} = \dot m v_b}}$$ MOM out of the CV



$$-F = \dot m V_b - \dot m V_J =$$

$$= \dot m (v_J - v_b)$$

$\dot m$ is still the $\underline{\text{same}}$ $= \rho A (v_J - v_b)$ $\Bigg\}$ $-F = -\rho A (v_J - v_b)^2$

always relative to the speed of the c.s.

So the result is the same independently of the choice of the ref. frame!

THAT'S COMFORTING (:

But the perspective change

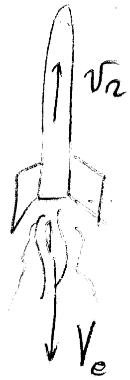
$\dot m$ is the same — defined with respect of the c.s. velocity

Momentum is not! v is relative

12/8

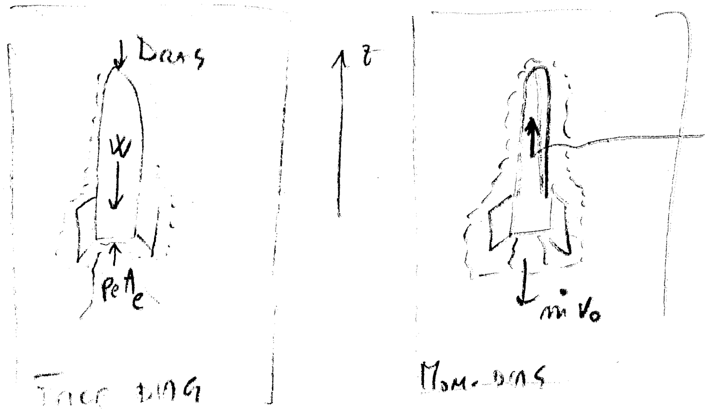
MOVING CONTROL VOLUME (Accelerating) — pp 231–232

Eq. motion of a rocket!
(no simplification)



exhaust gas V_e through nozzle A_e

Note c.v. → accelerate with the rocket
the C.V. is not inertial.



as the rocket is moving UPWARDS (accelerating) we have an ACCUMULATION/~~[illegible]~~ in MOMENTUM

$$\underbrace{\frac{d}{dt}(m_r V_r)}$$

$$\frac{d}{dt}\left[\int_{CV} v_z \rho \, dVol\right] = \frac{d}{dt}\left[v_r \int \rho \, dV\right] = \frac{d}{dt}(m_r v_r)$$

$$\Sigma F_z = p_e A_e - W - D$$

IN FACT THIS IS

$$\Sigma \dot m V_o = \dot m (v_r - V_e) \sim \dot m \left(-\underbrace{(V_e - v_r)}\right)$$

velocity with respect to a fixed ref. system (GROUND)

$$\Sigma \dot m_i V_i = 0$$

$$p_e A_e - W - D = \frac{d}{dt}(m_r V_r) + \dot m (v_r - V_e)$$

$$\dot m v_r + \frac{d}{dt}(m_r V_r) =$$

CONTINUITY

$$-\frac{d}{dt}(m_r)\cdot v_r + \frac{d}{dt}(m_r V_r) =$$

$$= m_r \frac{d}{dt} V_r$$

apply CONTINUITY; $\dot m_{IN} = 0$

$$\frac{d}{dt} m_r + \dot m = 0$$

the mass of the rocket is reducing while the rocket is accel. and pushing out fluid

$$m_r \frac{dV_r}{dt} = \overbrace{p_e A_e + \underset{\rho_e A_e V_e^2}{\dot m V_e}}^{\text{Thrust } T} - W - D$$

14) d Moment of MOMENTUM eq.

Torque acting on a fluid in a CV
induce changes in the ANG. MOM. of a fluid.

$$\underbrace{\sum \text{Moment}_{\text{ext}}} = \frac{d}{dt}\left(H_{sys}\right)$$

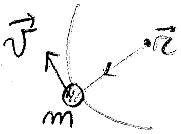
TOTAL ANGULAR MOMENTUM of a system

Lagrangian equat.

$$\sum F_{ext} = \frac{d}{dt}\left(\text{Mom}\right)_{sys}$$

MOMENTUM

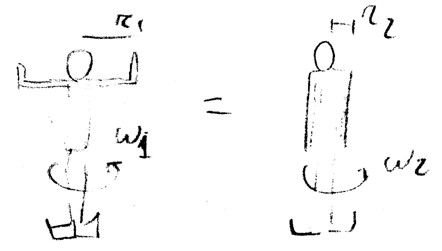
In analogy with we write



ANG. MOM $= \vec{r} \times m\vec{v}$

$$\frac{d}{dt}\left(H_{sys}\right) = \sum M_{EXT} = \frac{d}{dt}\int_{CV}\left(\bar{r}\times\bar{v}\right)\rho\, dV + \int_{CS} \bar{r}\times\bar{v}\rho\, \bar{V}\cdot dA$$

Rem. conserv. ANG MOM.



$$\underbrace{R_1 > r_2}$$

$$\omega_2 > \omega_1$$

or in its integral form

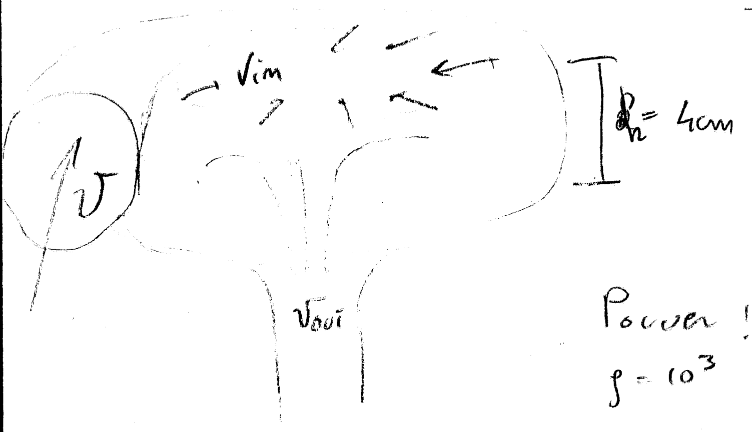
$$\sum M_{EXT} = \frac{d}{dt}\int_{CV} \bar{r}\times\bar{v}\,\rho\, dV + \sum_{CS} \vec{r}_o \times \dot{m}_o \vec{V}_o$$

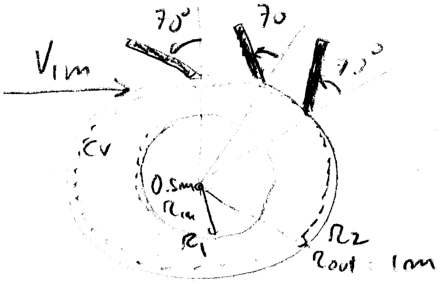
$$- \sum_{CS} \vec{r}_{in} \times \dot{m}_{in} \vec{V}_{in}$$

14e

Ex (6.9) BOOK

FRANCIS TURBINE
↳ picture online





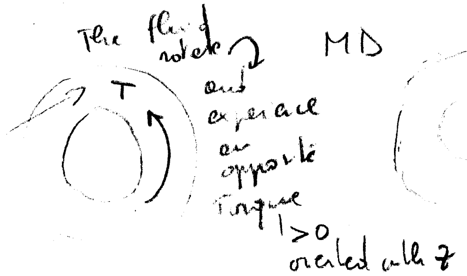
Power ! ?

$\rho = 10^3$

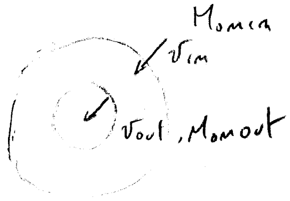
Given R_1, R_2 Q, ω

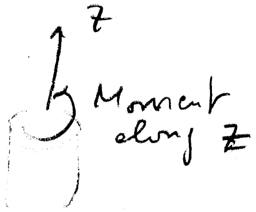
F.D.

MOMENT DIAGR.



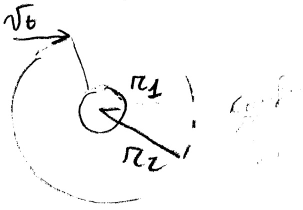
MD



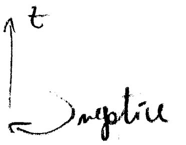


Let us write

$$\sum M_z = \frac{d}{dt}\int_{CV} (\bar{r} \times \bar{v}_z)\,\rho\, dVol + \sum_{CS} (r_{OUT} \times \dot{m}\, v_{out})_z - \sum_{CS} [r_{in} \times (\dot{m}\, v_{in})]_z$$



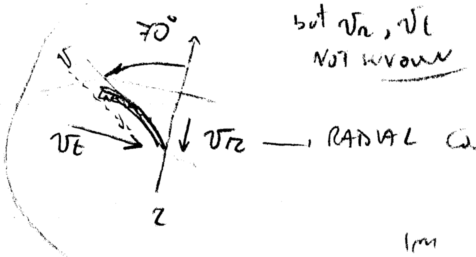
$\sum M_z = T$



steady $\frac{d}{dt}\int_{CV} = 0$

$\bar{v}_t \perp \bar{r}$ →

Inlet Mom → $\sum_{CS} r_i \times \dot{m} v_i \Big|_z = - \dot{m}\, r_2\, v_t$

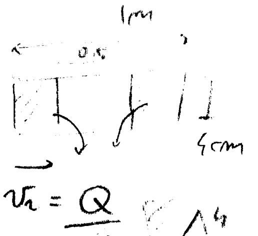


— RADIAL COMPON.

HP. $\bar{v}$ KNOWN but v_r, v_t NOT KNOWN

Outlet Mom → $\sum_{CS} (r_o \times \dot{m} v_o)_z = 0$

No moment along z



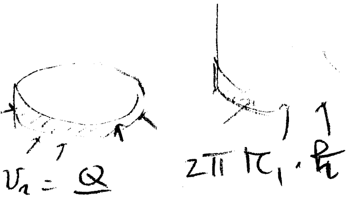
$v_r = \frac{Q}{\pi D \cdot h}$

So $T = \dot{m}\, r_o\, v_t = \underbrace{\rho Q}_{\dot{m}} \cdot r_o\, v_t$ ‖

r_o or r_2

‖ $P = T \cdot \omega$ ANG. vel.

$\frac{v_t}{v_r} = \tan 70°$

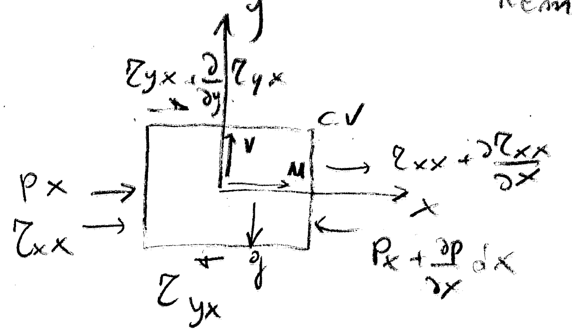


$v_r = \frac{Q}{\text{Area cyl}}$

$2\pi r_1 \cdot h$

4f

NAVIER Stokes Eq.



Rem. $\tau_{ij} = \frac{F_t \text{ in the J component}}{\text{Area with normal DIRECT oriented along } i}$

All forces along x

BODY F → $g_x \rho \Delta x \Delta y$

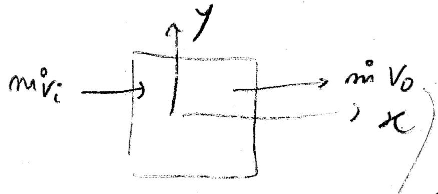
Fpress $F_{x,p} = \underbrace{\frac{\partial p}{\partial x} dx + p}_{+} \underbrace{- p}_{-} = -\frac{\partial p}{\partial x} \cdot dx$

$F_{\tau,x} = {}_{\text{NORMAX}} \frac{\partial \tau_{xx}}{\partial x} dx + {}_{\text{TANGENT}} - \frac{\partial \tau_{yx}}{\partial x}$

$$\frac{d}{dt}\int_{CV} \rho u \, dV_{OL} = \underbrace{\int_{CV} \frac{\partial}{\partial t}(\rho u)\, dV_{OL}} + \int_{CS} \rho u \, \underset{\substack{\downarrow \\ \text{velocity of the control surface} \; = 0}}{V_c} \cdot A$$

$= \frac{\partial}{\partial t}(\rho u)\, dx\, dy$

$\sum_{CS} \dot{m} \dot{V}_o$



$\overbrace{\rho dy \cdot v_x}^{\dot{m}_o} \cdot v_x = \rho dy v_x^2 = \rho v_x^2 dy$

$$\frac{\partial}{\partial t}(\rho u) + \frac{\partial}{\partial x}(\rho u u) + \frac{\partial}{\partial y}(\rho u v) = \rho \frac{\partial u}{\partial t} + \rho u \frac{\partial u}{\partial x} + \frac{\partial}{\partial y}(\rho u v)$$
$$+ u\left\{\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v)\right\}$$

$$\rho \frac{\partial u}{\partial t} + \rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \rho g_x$$

$\tau_{ij} = \mu \frac{du_i}{dx_j}$

15) a) The energy equation.

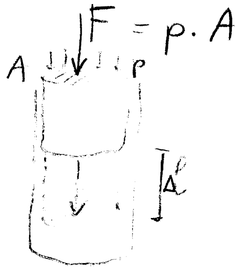
Fluid has energy in different forms

- gravitational energy → DAM, PUMPING STORAGE
- Kinetic energy (moving fluid) ↗
- thermal energy (hot steam, hot jet)

A fluid with ENERGY can be used to produce work.

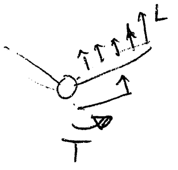
e.g a piston exerts a pressure
on a fluid that moves over a
finite distance ℓ

1)



$$\underset{\text{DONE BY THE PISTON}}{\text{Work}} = F \cdot \Delta\ell = p A \Delta\ell .$$

Vice versa the fluid may move a piston and produce energy work.



2) Wind passes over the blades of a turbine.
The force of the wind induce the blades to rotate → generates a Torque T that does work on the blades

Lift force with a ARM

ANGULAR DISPLACEMENT

$$\text{Work} = F \cdot \Delta\ell = T \cdot \Delta\theta$$

$$[W] = [E] = J = N \cdot m$$

other units are BTU
calory

Machine is any device that TRANSMITS or MODIFIES ENERGY

If I move up a block, I do a work, and the block change its potential energy.

fluid → Force (LIFT)
↓
Torque
↓
ROTATION

Work is done.

KONDZIOLKA
tabl. 2 → 6+8

$\boxed{15b}$

Power: rate of work / or rate of energy

$$P = \lim_{\Delta t \to 0} \frac{\Delta W \text{ or } \Delta E}{\Delta t} \;\longrightarrow\; P = P(t) \text{ INSTANT.}$$

$$P = \dot{W}$$

following this definition, we have that

ASSUME $\Delta x = \Delta l$ $$P = \lim_{\Delta t \to 0} \frac{F \Delta x}{\Delta t} = F \cdot V$$

Velocity of a moving body

or

$$P = \lim_{\Delta t \to 0} \frac{T \cdot \Delta \theta}{\Delta t} = T \cdot \omega$$

ANGULAR SPEED of the turbine

$[\omega] = \frac{RAD}{s}$ or RPM RPS

$[P] =$ WATT, base power } Import

$\frac{J}{s}$

note:

1 kWh = kW·h

$10^3 \cdot W \cdot h = \underbrace{P \cdot \text{time}} = \text{Energy}$

it's an integral of power in time

→ energy that is consumed

Eolos turbine

$P = 2.5$ MW

Nuclear Powerplant → reactor ~ 0.5 GW → 3÷4 GW

\[5c\] General equation of energy

1st LAW of Thermodynamics → applied on a fluid system

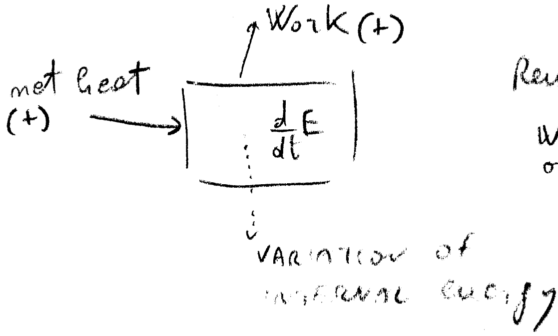
LAGRANGIAN view: we monitor the energy of the water parcels while we move with them

$$\dot{Q} - \dot{W} = \frac{dE_{sys}}{dt}$$

$\frac{dE_{sys}}{dt}$ → rate of change of the energy of the system

$\dot{Q}$: net rate of thermal energy entering in the system

$\dot{W}$: net rate at which the system does work on the surrounding environment



Rem. Q_{IN} +, Q_{out} −

Work done by the system on the SURROUNDINGS + (BOUND. CONDIT.)

Rem: the system is the fluid

→ turbine results in positive work, pump results in negative work

As usual we use the Reynolds transport theorem to transform the LAGRANG. equation in an Eulerian (fixed CONTROL VOLUME EQ.)

$$\dot{Q} - \dot{W} = \underbrace{\frac{d}{dt}\int_{CV} e\rho \, dVol + \int e\rho \, \overline{V} \cdot d\overline{A}}$$

$\frac{d}{dt}E_{sys}$ = VARIAT. of E in the C.V + net flux of E through the C.S.

$$e = e_K + e_p + u$$

e: energy per unit mass of the fluid

u: internal en. or thermal en.

e_p → $\frac{\text{Potential energy}}{\text{mass}} = \frac{mgz}{m} = gz$

e_K → $\frac{\text{Kinetic energy}}{\text{unit mass}} = \frac{mV^2/2}{m} = \frac{V^2}{2}$

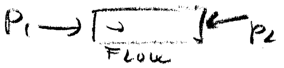
$\boxed{15d}$

General form of the energy equation

$$\dot{Q} - \dot{W} = \frac{d}{dt}\int_{CV}\left(\frac{V^2}{2} + gz + u\right)\rho\, dV_{ol} + \int_{CS}\left(\frac{V^2}{2} + gz + u\right)\rho\, \vec{V}\cdot d\vec{A}$$

Flow work → work done by pressure forces outside the c.v. → inlet (push flow in) — work done on the system < 0

outlet (resist the flow to move out) — work done by the system > 0

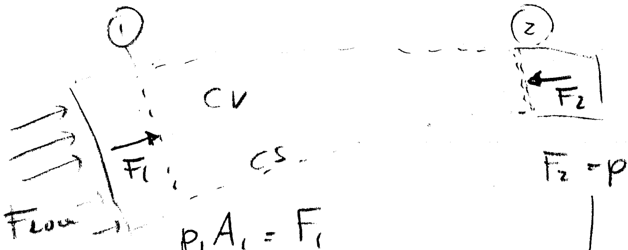


\+

Shaft work → work done by a mechanical device on the fluid — extract a / add energy to the fluid

$$\dot{W}_{shaft} = \underset{(>0)}{\dot{W}_{turbines}} + \underset{(<0)}{\dot{W}_{pump}}$$

Let us derive an equation for the flow work



$p_1 A_1 = F_1$

$F_2 = p_2 A_2$

as the fluid moves by Δx in $1\,\Delta t$, the work done by the pressure force (work done by the system) is $\Delta W_2 = F_2 \cdot \Delta x_2$ with $\Delta x_2 = V_2\,\Delta t$

so $\Delta W_2 = p_2 A_2 \cdot V_2\,\Delta t$

the rate of work $\dot{W}_2 = \lim\limits_{\Delta t \to 0}\frac{\Delta W_2}{\Delta t} = p_2 A_2 V_2 = \frac{p_2}{\rho}\cdot\overbrace{\rho A_2 V_2}^{\dot{m}_2}$

Thus $\dot{W}_2 = \dot{m}\frac{p_2}{\rho}$ + work done by the fluid.

Accordingly

$\dot{W}_1 = -\dot{m}\frac{p_1}{\rho}$

sign → ~~Δx~~ of work done by the env. on the fluid system

$\boxed{15e}$ From the derivation we have:

$$\dot{W}_{flow} = \dot{W}_2 + \dot{W}_1 = \underbrace{\dot{m}\frac{P_2}{\rho} - \dot{m}\frac{P_1}{\rho}}_{\text{in general}}$$

Remember CONTINUITY $\frac{dm}{dt} = 0 \Big\} \dot{m} = \text{const}$

$$= \underbrace{\sum_{OUTLETS} \dot{m}_{OUT}\frac{P_{OUT}}{\rho} - \sum_{INLETS} \dot{m}_{in}\frac{P_{in}}{\rho}}_{\text{even broader}}$$

$$= \int_{CS} \left(\frac{P}{\rho}\right) \cdot \underbrace{\rho\vec{V}\cdot d\vec{A}}_{\dot{m}}$$

So $\dot{W} = \dot{W}_{flow} + \dot{W}_{shaft} = \int_{CS} \frac{P}{\rho}\rho\bar{V}\cdot dA + \dot{W}_{Turb} - \dot{W}_{pump}$

If we consider the previous general form:

$$\dot{Q} - \dot{W} = \dot{Q} - \dot{W}_{shaft} - \dot{W}_{flow} =$$

$$= \dot{Q} - \dot{W}_{shaft} - \int_{CS} \frac{P}{\rho}\rho\bar{V}\cdot dA$$

$$= \frac{d}{dt}\int_{CV} \left(\frac{V^2}{2} + gz + u\right)\rho\, dV_{ol} + \int_{CS}\left(\frac{V^2}{2} + gz + u\right)\rho\bar{V}\cdot dA$$

$$\dot{Q} - \dot{W}_{shaft} = \frac{d}{dt}\int_{CV} \frac{V^2}{2} + gz + u\ \rho\, dV_{ol} + \int_{CS}\left(\frac{V^2}{2} + gz + \underbrace{\overline{\cancel{h}u + \frac{P}{\rho}}}_{}\right)\rho\bar{V}\cdot d\vec{A}$$

Combine the flux of energy through CS and the work done by the flow

where $u + \frac{P}{\rho} \overset{\cdot}{=}_{DEF}$ enthalpy

For a discretised system (INLET, OUTLET)

$$\dot{Q} - \dot{W}_S = \frac{d}{dt}\int_{CV}\left(\frac{V^2}{2} + gz + u\right)dV_{ol} + \sum_{CS}\dot{m}_o\left(\frac{V_o^2}{2} + gz_o + h_o\right) - \sum \dot{m}_{in}\left(\frac{V_i^2}{2} + gz + h_i\right)!$$

the pressure work is seen as a variation of enthalpy. REM V_{IN}, V_{OUT} const in the cross sec

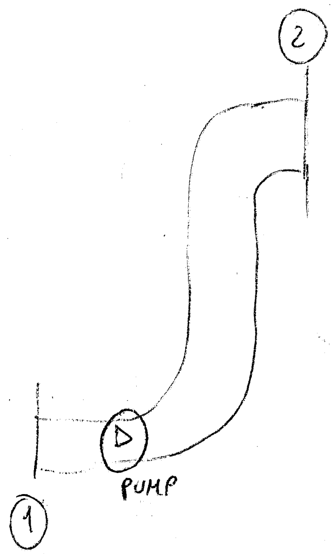
16 2) Steady case in a pipe flow → ~~[illegible] of turb. flows~~

with no heat flux INPUT

$$\dot{Q} - \overset{0}{\dot{W}}_s + \int_{A_1}\left(\frac{p_1}{\gamma} + \rho z_1 + u_1\right)\rho V_1\,dA_1 + \int_{A_1}\rho\frac{V_1^3}{2}\,dA_1 \longrightarrow \int_{A_1}\frac{V_1^2}{2}\cdot\rho V_1\,dA = \int\rho\frac{V_1^3}{2}dA$$

SPLIT THE Kin. energy term.

$$= \int_{A_2}\left(\frac{p_2}{\gamma} + \rho z_2 + u_2\right)\rho V_2\,dA_2 + \int_{A_2}\rho\frac{V_2^3}{2}\,dA_2$$



in a pipe cross section

we have $\frac{p}{g\rho} + z = h$ piezometric head constant across 1, 2

If we assume that temperature is constant in the cross section (well mixed flow) then (we can take out of the integral)

$$\dot{Q} - \overset{0}{\dot{W}}_s + \overbrace{\left(\frac{p_1}{\rho\cancel{\gamma}} + \rho z_1 + u_1\right)}^{\text{const.}}\cdot\overbrace{\int_{A_1}\rho V_1\,dA_1}^{\dot{m}} + \int_{A_1}\rho\frac{V_1^3}{2}\,dA_1$$

$$= \underbrace{\left(\frac{p_2}{\rho} + \rho z_2 + u_2\right)}_{\text{CONSTANT}}\underbrace{\int_{A_2}\rho V_2\,dA_2}_{\dot{m}} + \int_{A_2}\rho\frac{V_2^3}{2}\,dA_2$$

IMPORTANT.

if we express

$$\int_{A_1}\rho V_1\,dA_1 \overset{\text{CONTINUITY}}{=} \dot{m} = \int_{A_2}\rho V_2\,dA_2$$

then we can also express

$$\int\rho\frac{V_1^3}{2}\,dA_1 = \frac{\overset{\text{Kin. en. correction}}{\alpha_1}\bar{V}_1^2}{2}\cdot\underbrace{\int\rho V_1\,dA}_{\dot{m}} = \dot{m}\,\alpha_1\frac{\bar{V}_1^2}{2}$$

16b) Let us put the kin. en. contribution aside:

$$\dot{Q} - \dot{W}_s + \left(\frac{p_1}{\rho} + g z_1 + u_1 + \alpha_1 \frac{\bar{V}_1^2}{2}\right)\dot{m} = \left(\frac{p_2}{\rho} + g z_2 + u_2 + \alpha_2 \frac{\bar{V}_2}{2}\right)\dot{m}$$

Rem.

$\dot{W}_{shaft} = \dot{W}_{turbine} - \dot{W}_{pump}$; divide by $\dot{m} g$

$$\underbrace{\frac{\dot{W}_p}{\dot{m} g}}_{h_p} + \frac{p_1}{\gamma} + z_1 + \alpha_1 \frac{V_1^2}{2g} = \underbrace{\frac{\dot{W}_t}{\dot{m} g}}_{} + \frac{p_2}{\gamma} + z_2 + \alpha_2 \frac{\bar{V}_2^2}{2g} + \frac{u_2 - u_1}{g} - \frac{\dot{Q}}{\dot{m} g}$$

h_p pump ~~turbine~~ head

turbine head

$$h_t \Longrightarrow \frac{\Delta W}{\cancel{\Delta t}} \cdot \frac{\cancel{\Delta t}}{\Delta m \; g} = \frac{\Delta F \cdot \Delta l}{\Delta m \cdot g} \sim \frac{\cancel{\Delta m} \cdot \cancel{acc.} \cdot \Delta l}{\cancel{\Delta m} \cdot \cancel{g}} \sim [l]$$

ok dimens.

$$\underbrace{\frac{p_1}{\gamma} + \alpha_1 \frac{\bar{V}_1^2}{2g} + z_1 + h_p = \frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_t}_{\text{mechanical energy}} + \underbrace{\left[\frac{1}{\rho}\overset{>0}{(u_2 - u_1)} - \overset{<0}{\frac{\dot{Q}}{\dot{m} g}}\right]}_{\text{thermal en}}$$

thermal en → always > 0 (~~[illegible]~~ ~~[illegible]~~ ~~heated~~ → friction, head loss caused by viscous mechanism (II LAWS of thermodynamics on PROCESS IRREVERSIBILITY)

h_L head losses.

$$\underbrace{\left(\frac{p_1}{\gamma} + \alpha_1 \frac{\bar{V}_1^2}{2g} + z_1\right)}_{} + h_p = \left(\frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2\right) + h_t + h_L$$

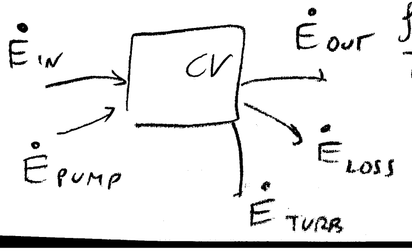
head carried by the flow IN + PUMP = head carried OUT + TURBINE + LOSSES (including friction)

ASSUMPT.: steady flow, pipe flow

1 INLET, 1 OUTLET

ρ = CONST

Temp., u CONST on the cross sect.



16c)

Energy equation vs. Bernoulli equat.

En. $$\alpha_1 \frac{\bar{V}_1^2}{2} + \frac{p_1}{\gamma} + z_1 + h_p = \alpha_2 \frac{\bar{V}_2^2}{2} + \frac{p_2}{\gamma} + z_2 + h_t + h_L$$ } PIPE f

Bern. $$\frac{V_1^2}{2} + \frac{p_1}{\gamma} + z_1 = \frac{V_2^2}{2} + \frac{p_2}{\gamma} + z_2$$ } ALONG A STREAMLINE

COMPARE...

Bernoulli EQ

MECHANICAL ENERGY

$\vec{F} = m\vec{a}$

along a streamline and integrated

ASSUMPT.

STEADY

INCOMPR.

INVISCID

Energy EQ

MECH & THERMAL ENERGY

↓

ASSUMPT.

STEADY

INCOMPR.

VISCOUS

assuming NO TURB, NO PUMP and a pipe as an inviscid streamtube with CONSTANT velocity in the cross sect. then,

$\alpha = 1$

$h_L = 0$

Energy ⇝ Bernoulli

but not viceversa

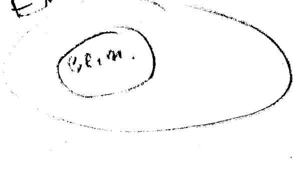
Big problem is how to calculate HEAD LOSS

resistance,

friction

geometry changes

energy dissipation



16d]

Power equation

Pump power

$$\underbrace{\dot{W}_p}_{\text{definition of } h_p} = \dot{m}\, g\, h_p = \rho \cdot Q \cdot g\, h_p = \gamma Q h_p$$

Similarly

$$\dot{W}_t = \dot{m}\, g\, h_t = \gamma Q h_t$$

In general we can express $\dot{W}$ as Power $\boxed{P = \gamma Q h}$ → both power IN — given to the fluid (PUMP)

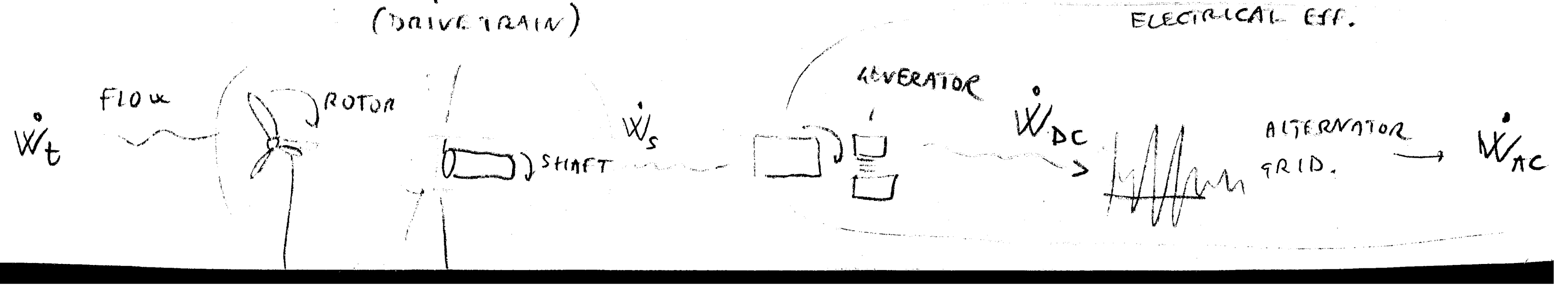
OUT — extracted from the fluid motion (TURB)

Efficiency

$$\eta = \frac{\text{Power OUTPUT}}{\text{Power INPUT}}$$ in any $\overset{\text{steps in the}}{\vee}$ transformation of energy

often we deal with **mechanical efficiency**:

for a turbine $$\underset{\text{Power OUTPUT of the rotating shaft (DRIVETRAIN)}}{\dot{W}_s} = \eta_t \cdot \underbrace{\dot{W}_t}_{\text{power input to the turbine from the flow}}$$



⑰ Energy grade line., Hydraulic grade line

Energy GRADE Line ; EGL
line that indicates the total
head at each location in a pipe system

$$EGL = \underbrace{\frac{\alpha V^2}{2g}}_{\text{velocity head}} + \underbrace{\frac{p}{\gamma}}_{\text{pressure head}} + z = \text{Total head } [m]$$

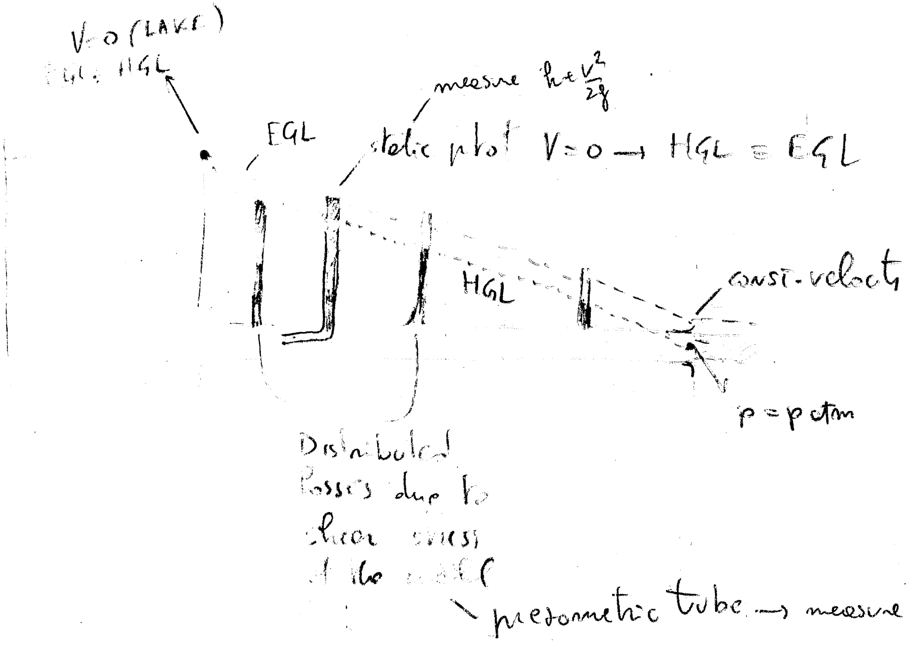
$\frac{m^2}{s^2} \cdot \frac{1}{\frac{m}{s^2}} = m$

$p = \gamma h$ so $\frac{p}{\gamma} = h$ [m]

z → elevation OR POTENTIAL HEAD

$$HGL = \frac{p}{\gamma} + z$$ = piezometric head (so no velocity head)

↓
if vel. = 0
then HGL ≡ EGL



measure $h + \frac{V^2}{2g}$
static pitot V=0 → HGL ≡ EGL

const. velocity, p = PATM, z = const

p = patm

Distributed losses due to shear stress at the wall

piezometric tube → measure $h = z + \frac{p}{\gamma}$

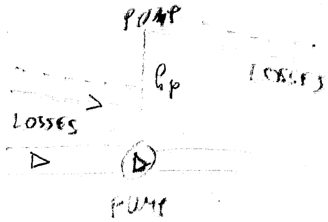
17b)

Some guidelines

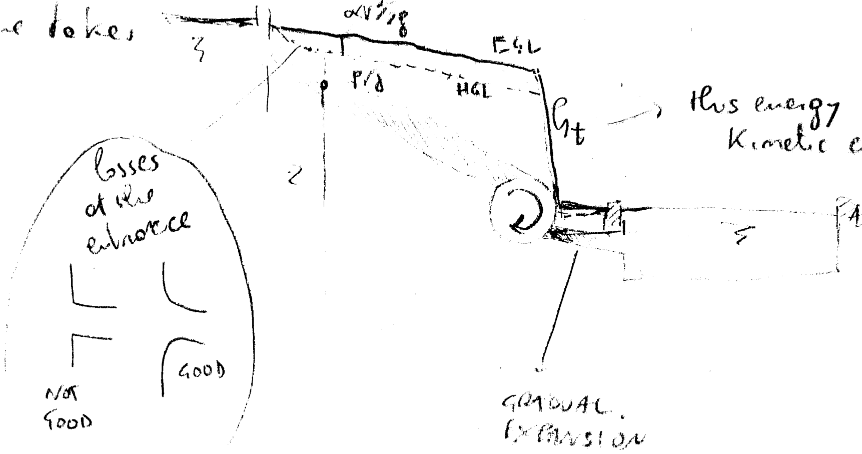
1) LAKES, TANKS $V=0 \rightarrow$ EGL = HGL

2) PUMP gives energy to the fluid, h_p applies to both EGL and HGL

in the form of a pressure head



3) Turbine takes energy

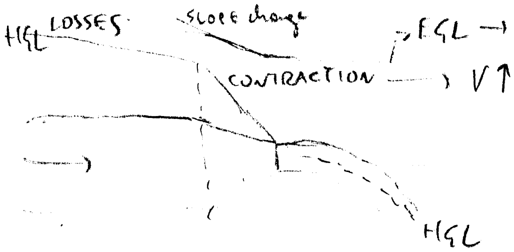


→ this energy would be Kinetic energy if the outlet was open with no turbine

GRADUAL EXPANSION → prevents energy losses

KIN. EN. EGL tends to the HGL

4) Discharge as a free flow



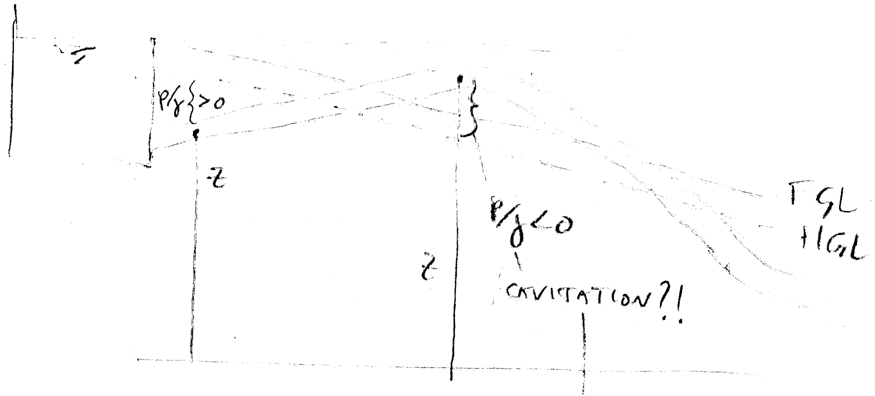
EGL → as V↑ in the contract, dissipation increases, so we have a drop
then, EGL ~ CONST as the Jet expands in the air
(p = ATM const.
z↓
Vel↓ } slowly it does decrease

CONTRACTION → V↑ p↓ so HGL↓

5) change in geometry → CONTINUITY change in velocity → with larger velocity, E will be dissipated faster (change in slope in the EGL

7c)

The case of negative pressure



} distributed losses, no change in geometry
slope = const.

it's gage pressure,
so $P_{gage} < 0 \to p < P_{ATM}$.

POWER POINT

18A) OLD → PPT ok 2013 spring

Dimensional ANALYSIS

1) How do we compare LABORATORY EXPERIMENT with REAL SCALE / FIELD SCALE PROBLEMS? } → e.g. pressure drop in a pipe flow due to distributed head losses.

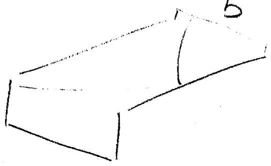
2) How do we seek for "universal solutions" in canonical flows? → JETS
WAKES
BOUNDARY LAYERS
PIPE FLOWS

Results must be presented in dimensionless form:

POWERPOINT

e.g. sediment transport:

BARS → L_{BARS} scale with channel width



$$\frac{L_{BARS}}{b} \sim \text{CONSTANT} \rightarrow \left.\frac{L_{BARS}}{b}\right|_{\substack{\text{field}\\ \text{RIVER}}} \approx \left.\frac{L_{BARS}}{b}\right|_{\substack{\text{LAB}\\ \text{expo.}}}$$

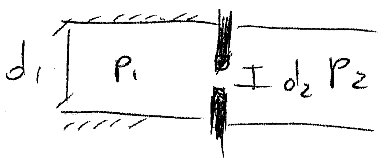
DUNES scale with river depth →

$$\left.\frac{L_{DUNES}}{\text{Depth}}\right|_{\text{field}} \sim \left.\frac{L_{DUNES}}{D}\right|_{\text{LAB.}} \quad \|$$

Typically we divide our $\underbrace{\text{quantity of interest}}_{\text{height or length of the bedform}}$ by a NORMALIZATION quantity – with the same dimension [width, or depth]

18 B) Let us go back

on the pressure drop in pipe flows → case of an abrupt contraction



$$\Delta p \propto \,? \begin{cases} d_1 d_2 & \text{pipe geometry} \\ \rho, \nu & \text{fluid properties} \\ v & \text{flow field} \end{cases}$$

Bernoulli suggest, velocity, density

$$\frac{v^2}{2g} + \frac{p}{\gamma} + z = \text{CONST}$$ BERNOULLI

Experimentally it is found that

$$\frac{\Delta p}{\frac{\rho V^2}{2}} = f\left(\frac{d_1}{d_2}, \frac{\rho V_1 d_1}{\mu}\right)$$! wow

how did they come up with this?

~ Bernoulli

$$\frac{\rho V^2}{2} = \frac{kg}{m^3} \cdot \frac{m^2}{s^2} = \frac{kg}{m s^2}$$

$$\Delta p = \frac{F}{A} \simeq \frac{kg\, m}{s^2} \cdot \frac{1}{m^2} = \frac{kg}{m s^2}$$

{ ok $\frac{\Delta p}{\frac{\rho V^2}{2}}$ is a NUMBER

this is more complicate than the previous case

$\frac{L_{BARS}}{L_{RIVER}}$

$\frac{d_1}{d_2}$ makes sense!

as $d_2 \downarrow$, $\frac{d_1}{d_2} \Uparrow$ more dissipation and flow DISTORTION more turbulence,

$\Delta p \uparrow$ $\overset{EGL}{\text{drop}}$! losses!

18c)

How about $\longrightarrow \rho \frac{V_1 d_1}{\mu}$?

$\rho \frac{V_1 d_1}{\mu}$ = Reynolds number $\overbrace{\frac{kg}{m^3} \cdot \frac{m}{s} \cdot m}^{\rho V d} : \frac{kg}{m s}$ ok

$\mu = \frac{F/A}{\frac{\Delta v}{\Delta y}} = \frac{N/m^2}{\frac{m}{s} \cdot \frac{1}{m}} \sim \frac{N \cdot s}{m^2} \sim kg \frac{m}{s^2} \cdot \frac{s}{m^2} = \frac{kg}{m s}$

$\tau = \mu \frac{dV}{dy}$

What is the Reyn. number? $\longrightarrow Re = \frac{V_1 d_1}{\nu}$ ← kin visc. $\frac{\mu}{\rho}$

$Re \uparrow$ as $\underbrace{U\uparrow, d\uparrow}$ $\nu\downarrow$

big fast pipe flow
of low viscosity liquid
|
as opposed to
small pipe with honey (viscous)

Re governs the
TRANSITION between LAMINAR and turbulent flows.

It makes sense that

$\Delta P_{LOSSES} \uparrow$ as the flow is more turbulent
|
more dissipative
messy flow!

18e

COMMON AND USEFUL DIMENSIONLESS NUMBERS

Given L lengthscale $[m]$
T timescale $[s]$
M mass $[kg]$

we can define

$$[V] = \frac{L}{T}$$

$$[\rho] = \frac{M}{L^3}$$

$$[\mu] = \frac{M}{LT}$$

$$[\sigma] = \frac{M}{T^2} \rightarrow \frac{N}{m} = \frac{M\overset{a}{L}}{T^2}\frac{1}{L} = \frac{M}{T^2}$$

$$[p] = \frac{N}{m^2} = \frac{M}{LT^2} = [\tau]$$

} dimensionless groups !

$$\left.\begin{aligned}[p] &= \frac{M}{LT^2} \\ \rho &= \frac{M}{L^3}\end{aligned}\right\} \underbrace{\frac{p}{\rho} = \frac{1}{T^2}L^2 = \left[\frac{L^2}{T^2}\right]}$$

$$\frac{p}{\rho V^2} = \text{number !}$$

same for τ (dimensionless)

$$C_p = \frac{p - p_0}{\frac{1}{2}\rho V^2}$$ pressure coefficient

$$C_f = \frac{\tau}{\frac{1}{2}\rho V^2}$$ shear stress coeff

$$\frac{V}{c} = \text{Mach number}$$ - $V > c$ compressible effects matter

c speed of sound

Reynolds number, Froude number

07

Let us define the Reynolds number

|

Bernoulli: in pipe z = const

$$\frac{p}{\gamma} + \frac{V^2}{2g} = \text{const}$$

$$\underbrace{\frac{p}{\rho} + \frac{V^2}{2}}\sim \text{const}$$

so $p \propto \rho V^2$ PUMP → increase pressure → move the fluid.

In terms of force → $F \sim p \cdot A \sim \rho V^2 \underbrace{L^2}_{\text{Area}}$ ok

The force resisting to motion is

$$\left.\begin{aligned} F_v &\sim \tau \cdot A \sim \tau L^2 \\ \tau &\sim \mu \frac{dV}{dy} \sim \mu \frac{V}{L} \end{aligned}\right\} F_v \sim \mu \frac{V L^2}{L} \sim \mu V L$$

$$\frac{\text{The inertial or kinetic force}}{\text{the viscous force}} = \frac{\rho V^2 L^{\cancel{2}}}{\mu \cancel{V} \cancel{L}} \sim \frac{\rho V L}{\mu} = Re = \frac{VL}{\nu}$$

what does it mean

high Re → V high AND OR L high

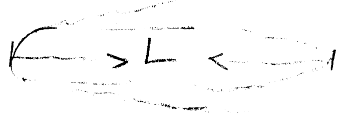
given a fluid property (ρ, ν, μ)

{ small Re, slow motions → BIOLOCOMOTION
small objects → DROPLETS MICRO ORGANISMS

MOVING SHIP → large L

OIL PIPE

TIDAL FLOW — L very big



Atm. boundary layer, planetary B.L.

$\nu\uparrow$ $\delta\uparrow$

18 i)

So for reproducing correctly the effect of gravity, dynamic similarity requires that $Fr_{model} = Fr_{real}$

(FROUDE π SIMILARITY)

If we want to reproduce the inertia to viscous forces → we must consider the Reynolds number

same logic of before

$$\frac{M_m a_m}{M_r a_r} = \frac{F_{viscous\,m}}{F_{viscous\,r}} \quad \text{const}$$

$$Re_m = Re_r$$

Complete similarity (: is achieved when we have GEOMETRIC & DYNAMIC SIMILITUDE ⟹ } COMPLETE SIMILARITY — ALL Π_i are involved $\Pi_m = \Pi_r$? | impossible !

~~Often~~ This is not possible we have to choose what to match ⟹ which forces can we match or better which force/ratio can we match?

C_f , C_p , Fr , Re

IMPORTANT : what is the key force?

gravity driven fluid → Fr. numbers (free surface flows)

flows with no free surface (e.g. parallel channel, or pipe flows) Reynolds number.

ok for SIMILIT. SLIDES DAY 18

ii) But the question remains?
How did we obtain this expression?

Buckingham theorem.

$$y_1 = f(y_2, y_3 \dots y_n) \quad n \quad \text{VARIABLES.}$$

can be rearranged into

$$\Pi_1 = f(\Pi_2, \Pi_3 \dots \Pi_{n-m})$$

where Π is a DIMENSIONLESS GROUP (number)
and m is the number of BASIC DIMENSION

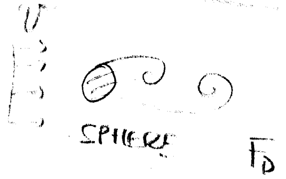
l, t, K_g

KINEMATIC

DYNAMIC $K_g \cdot \frac{l}{t^2}$ = Newton

Exponent method.

DRAG OF A SPHERE



$$F_D = f(V, \rho, \mu, D) \quad \rightarrow L$$

$$F = \frac{ML}{T^2} \quad \frac{L}{T} \quad \frac{M}{L^3} \quad \frac{M}{LT}$$

$n = 5$ VARIABLES
$m = 3$ DYNAMIC PROBL.
} $n - m = 2$ DIM. GROUP

let me write

$$[F_d] = \frac{ML}{T^2} = \left(\frac{L}{T}\right)^a \cdot \left(\frac{M}{L^3}\right)^b \cdot \left(\frac{M}{LT}\right)^c (L)^d$$

18) e

$M^1 = M^1$ → exponents

$1 = b + c$

$L = L^1$

$1 = a - 3b - c + d$

$\frac{1}{T^2} \rightarrow T^{-2}$

$-2 = -a - c$

$$\left\{\begin{array}{l} b + c = 1 \\ a - 3b - c + d = 1 \\ a + c = 2 \rightarrow \end{array}\right\} \quad \begin{array}{l} b = 1 - c \\ a = 2 - c \\ 2 - c - 3 + 3c - c + d = 1 \\ (d = 2 - c) \end{array}$$

Let us write

$$F = V^{2-c} \cdot \rho^{1-c} \mu^c D^{2-c} = V^2 \rho D^2 \cdot \frac{\mu^c}{V^c \rho^c D^c}$$

$$\underbrace{\frac{F}{\rho V^2 D^2}}_{\Pi_1} = \Big(\underbrace{\frac{\mu}{\rho V D}}_{\Pi_2}\Big)^c$$

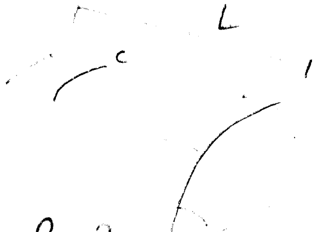
$$\underbrace{\left(\frac{F}{\rho V^2 D^2}\right)}_{\Pi_1} = f\Big(\underbrace{\frac{\rho V D}{\mu}}_{\Pi_2}\Big)$$

18) g very important

$\downarrow$

define velocity and lengthscale correctly

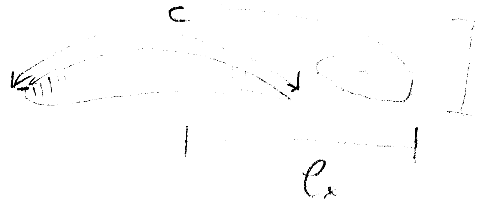
e.g



plane:
what is the
key lengthscale?

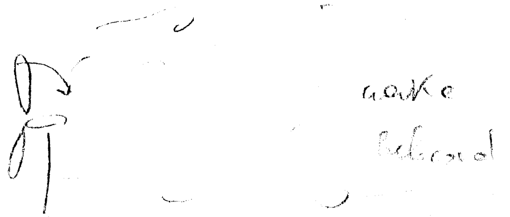
L blade

c chord length



l_y relevant scale

For a turbine?



} $L \sim d_{ROTOR} \sim$ blade scale not chord.

Other important number e.g. open channel flow

FROUDE Number

$$\frac{\text{KINETIC / INERTIA FORCE}}{\text{GRAVITY FORCE}} = \frac{\rho V^2 L^2}{\rho L^3 g} = \frac{V^2}{Lg} \rightarrow \frac{V}{\sqrt{Lg}}$$

$$\llcorner \, M \cdot g = \rho \underbrace{L^3}_{V} \cdot g$$

what is L?
depth of a river

Fr ↑ fast flow MOUNTAIN RIVER

Fr < 1 slow flow meandering river in flat terrain } still both gravity DRIVEN

18) J Some examples

Testing a valve for a real scale pipe → No free surface flow → No gravity → match Re

$Q = 700$ cfs

$L = 6$ ft

$$\boxed{\begin{matrix}\text{Model } L_m = 1\text{ ft} \\ Q_m\ ?\end{matrix}}$$

$Re_m = Re_r$

$\frac{V_m L_m}{\nu_m} = \frac{V_r L_r}{\nu_r}$ ... same fluid $\nu_m = \nu_r$

So $\frac{V_m}{V_r} = \frac{L_r}{L_m}$, $Q = V \cdot A = V \cdot L^2$

$$\frac{Q_m}{Q_r} = \frac{V_m L_m^2}{V_r L_r^2} = \frac{L_r}{L_m} \cdot \frac{L_m^2}{L_r^2} \qquad \frac{L_m}{L_r} = \frac{1}{6}$$

So $Q_m = \frac{Q_r}{6} = \frac{700}{6} \approx 117$ cfs.

check: $\frac{1}{6} = \frac{V_m}{V_r} \cdot \left(\frac{L_m}{L_r}\right)^2 = \frac{V_m}{V_r} \cdot \frac{1}{36}$

So $\frac{V_m}{V_r} = 6$

$V_m = 6 \cdot V_r$

higher velocity in the model and same Re!

2)

Drag force in a wind tunnel

object

$L_m = \frac{1}{10} L_r$

DYNAMIC SIMILITUDE & GEOMETRIC

$F_m = 1530\,N$ DRAG Measured.

what is F_{real}? → F_{REAL}!

Re similarity, geom. similarity → same Π_i

$Re_m = Re_r \rightarrow \frac{V_m}{V_r} = \frac{L_r}{L_m}$

$$\frac{F_r}{\frac{1}{2}\rho_r V_r^2 L_r^2} = \frac{F_m}{\frac{1}{2}\rho_m V_m^2 L_m^2} \; ; \; \frac{F_r}{F_m} = \frac{\cancel{\frac{1}{2}}\rho_r V_r^2 L_r^2}{\cancel{\frac{1}{2}}\rho_m V_m^2 L_m^2} = 1 \longrightarrow \underline{F_m = F_r}$$

same forces

18 e)

Spillway models

↓

the key DIMENSIONLESS # is Fr

↓

The best option is
to keep same Fr
but make sure we are
in the Reynolds INDEPENDENT REGIME !
(Big models)

$$\frac{L_m}{L_r} = \frac{1}{50}$$

$Q_r = 15\,000\ m^3/s$

Q_m ?

$$Fr_m = Fr_r \longrightarrow \frac{V_m}{\sqrt{g L_m}} = \frac{V_r}{\sqrt{g L_r}} \longrightarrow \frac{V_m}{V_r} = \sqrt{\frac{L_m}{L_r}}$$

if I measure $V = 1\ m/s$ in the model

$$\downarrow$$

$$V_r = \frac{V_m}{\sqrt{\frac{1}{50}}} = V_m \sqrt{50} = 4.3\ m/s$$

real

$$\frac{Q_m}{Q_r} = AV = \frac{L_m^2 V_m}{L_r^2 V_r} = \frac{L_m^2}{L_r^2} \cdot \frac{L_m^{1/2}}{L_r^{1/2}} = \left(\frac{L_m}{L_r}\right)^{5/2}$$

if $\frac{L_m}{L_r} = \frac{1}{50}$ $\quad Q_m = Q_r \left(\frac{1}{50}\right)^{5/2} \sim 1\ m^3/s$

DONE ALREADY

Note that

$$\left.\begin{array}{ll} Fr \longrightarrow & \frac{V_m}{V_r} = \sqrt{\frac{L_m}{L_r}} \\ \\ Re & \frac{V_m}{V_r} = \frac{L_r}{L_m} \end{array}\right\} \text{impossible to match !!}$$

GRAVITY, KIN and
VISCOUS forces
cannot be all scaled up.
↳ Fr ok! in the INDEP. regime !

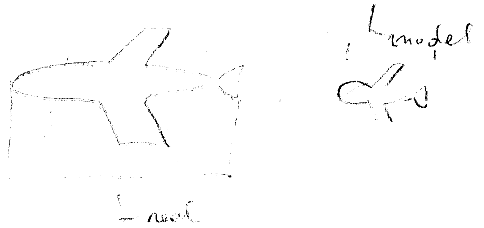
18) l_1

SIMILITUDE → predict performance based on a scaled model

Model design, construction, experiments & testing

①
Geometrical similitude
↓
exact replica

$$\frac{l_{model}}{l_{real}} = L$$



note that

$$\rho L^2 \frac{V}{t} = \rho \frac{L^3 V}{\frac{t}{V}} \cdot \rho L^2 \text{ [illegible]}$$

$= \frac{t}{V}$

②
Dynamic similitude

$$\frac{F_m}{F_{real}} = \text{const} \longrightarrow \frac{m_m \cdot a_m}{m_r \cdot a_r} = \frac{(\rho L^3 V/t)_m}{(\rho L^3 V/t)_r}$$

forces acting on the fluid are similar

if we are talking about gravity forces. then

e.g. for HYDRAULIC STRUCTURES OR RIVERS

BLACKBOARD 18 l

$$\frac{F_{g\,m}}{F_{g\,r}} = \frac{\rho L^3 \cdot g)_m}{\rho L^3 g)_r} = \frac{\gamma_m L_m^3}{\gamma_r L_p^3}$$

This below HAS TO BE SATISF.

$$\frac{F_m}{F_{real}} = \frac{F_{g\,m}}{F_{g\,real}}$$

SAME FOR Re

$$\rho L^3 \frac{V}{t} = \mu V L \sim \tau A \quad \mu \frac{dU}{dy} \cdot A$$

$$\frac{\rho L^3 V/t)_m}{\rho L^3 V/t)_r} = \frac{\gamma_m L_m^3}{\gamma_r L_r^3} \longrightarrow \underbrace{\frac{V_m}{t_m g_m} = \frac{V_r}{t_r g_r}}$$

but $t = \frac{L}{V}$

$$\frac{V_m^2}{g_m L_m} = \frac{V_r^2}{g_r L_r} \vdash \boxed{F_r = F_{r\,m}}$$

WITH DYNAMIC & GEOM. similitude the Π_i GROUPS are the same and the forces are the SAME → NOT ALL forces ... the ... the RATIO $F_m/F_g = \frac{F_r}{F_{g\,r}}$

18) K

Do we have to be at the exact same Re to achieve dynamic similitude ?

$\Pi_{model} = \Pi_{real}$?

In many situation

$$\frac{F}{\frac{1}{2}\rho V^2} \quad \text{or} \quad \frac{\Delta p}{\frac{1}{2}\rho V^2}$$

are Reynolds number independent, providing Re exceeds certain values

↳ if $\frac{\text{Kinetic force}}{\text{viscous}} > 10^3$ is fine ($= 10^5$ or 10^6 no difference)

SKIP

SKIP.

e.g.

CAR test in a wind tunnel $\left\{ \begin{array}{l} L = 4 \\ V = 100\,km/h \end{array} \right.$

$$Re_r = \frac{VL}{\nu} = \frac{\overset{27\,m/s}{100\,km/h} \cdot 4\,m}{1.5\,\frac{m^2}{s}} - 7.4\,10^6$$

ν

$$\left.\frac{F_D}{\frac{1}{2}\rho V^2}\right\} = \underbrace{C_{DRAG\,m}} = C_{DRAG\,real} \text{ if } Re_{model} > 10^5 \rightarrow \frac{V_m L_m}{\nu} > 10^5$$

the Force is different but the DRAG coeff is the same!

$$V_{model} \geq \frac{10^5 \cdot \nu}{L_m} \; ; \; \geq \frac{10^5 \cdot 10^{-5}\, 1.5}{L_m} = \frac{1.5}{L_m}$$

will I have the same Re? NO.

Will I have the same force? NO

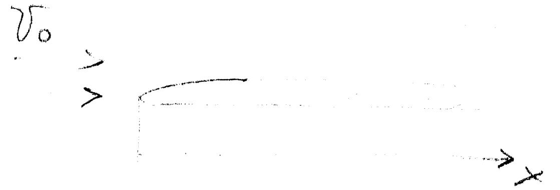
But I CAN SCALE the force!

So if $L_m = \frac{1}{10} L_r = 0.4\,m \rightarrow V_m > 3.8\,m/s$

18)e LAMINAR
Boundary Layer

PRANDTL 1904 → Viscous effect are concentrated close to the surface

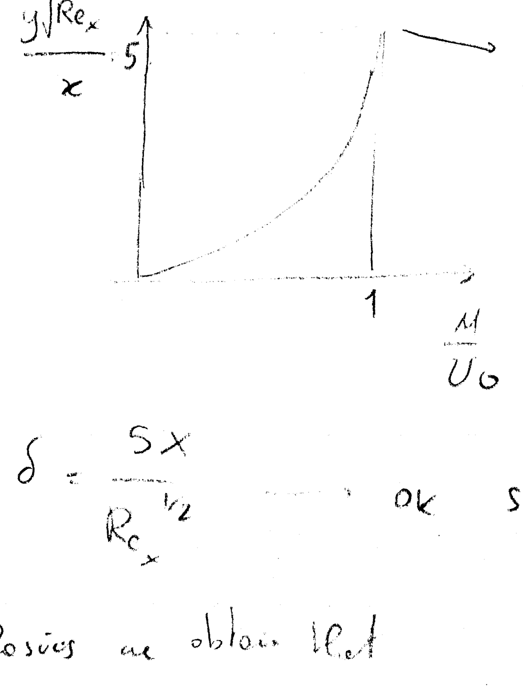
BLASIUS 1908 → NUMERICAL solution for LAM B.L.



$$Re_x = \frac{U_0 x}{\nu} < 500'000 \text{ (Exp. observ)}$$

δ defined such that $\frac{u}{U_0} = 0.99$

From the solution of Blasius



→ IMPORTANT! This point is on a DIMENSIONLESS GRAPH!

subst. $y = \delta$

$$\frac{\delta\sqrt{Re_x}}{x} = 5 \longrightarrow \delta = \frac{5x}{Re_x^{1/2}}$$ OK so $\delta = \delta(x)$ correct!

Also from Blasius we obtain that

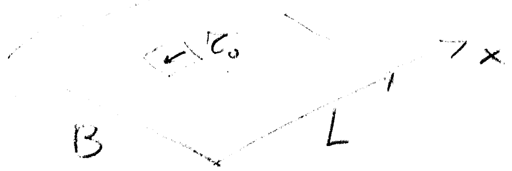
$$\left.\frac{du}{dy}\right|_{y=0} = 0.332\,\frac{U_0}{x} Re_x^{1/2} = 0.332\,\frac{U_0^{3/2}}{x^{1/2}\nu^{1/2}} \Bigg\}\ \tau_0 = 0.332 \cdot \mu \cdot \frac{U_0}{x} Re_x^{1/2}$$

So as $x \nearrow$ (moving along the plate), $\left.\frac{du}{dy}\right|_{y=0} \downarrow$ and thus $\tau_0 \downarrow$ $\qquad \delta \uparrow$

viscous DIFFUSION OF MEAN SHEAR!

(5) f)

Let us now consider
the full DRAG force acting on a plate



$$\underset{\text{shear}}{F_s} = \int_0^L B \cdot \tau_0 \, dx$$

with $\tau_0 = 0.332 \frac{V_0}{x} \cdot \frac{V_0^{1/2} x^{1/2}}{\nu^{1/2}} \cdot \mu$

$$\text{So} \quad F_s = \int_0^L B \, \tau_0 \, dx$$

$$F_s = 0.664 \, \mu V_0 Re_L^{1/2} \quad \text{where } Re_L = \frac{L \cdot V_0}{\nu}$$

Rem: dimensionless groups.

I DID NOT SPECIFY LAM OR TURB. FLOW

$$\overset{\text{small c}}{C_f} = \frac{\tau_0}{\rho V_0^2/2} = \frac{0.332 \frac{V_0}{x} \cdot \frac{V_0^{1/2} x^{1/2}}{\mu^{1/2}} \cdot \mu \cdot \rho^{1/2}}{\rho \frac{V_0^2}{2}} = \frac{0.664}{Re_x^{1/2}}$$

shear stress coeff

If we consider the full force

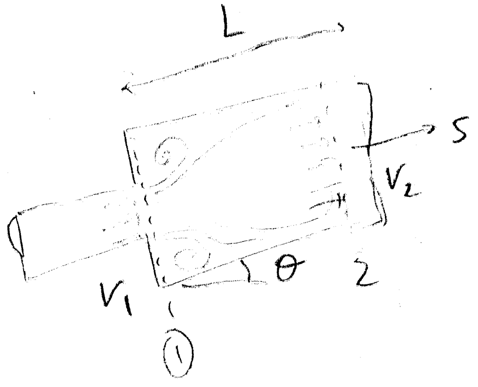
$$\text{BIG } C \leftarrow C_{F_{ORCE}} = \frac{F_s / \overset{BL}{A}}{\rho V_0^2/2} = \frac{1.33}{Re_L^{1/2}}$$

So as L ↑, F_s ↓ ... UNTIL LAM. flow gets unstable → TURB.

21(A)

Review

Abrupt expansion

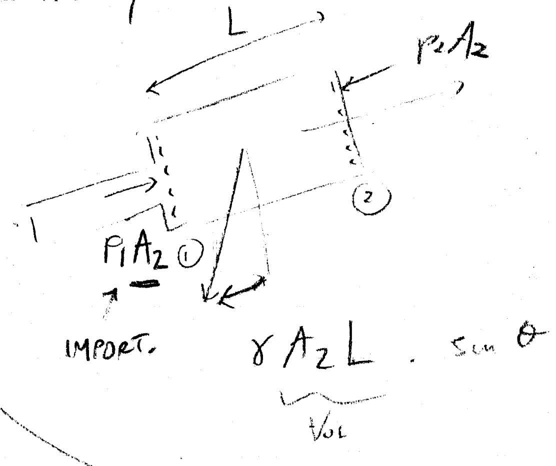


$$\text{✯)}\quad \frac{p_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \boxed{\alpha_2} \frac{V_2^2}{2g} + z_2 + \boxed{h_L} \qquad \text{Energy eq.}$$

assume turb. flow $\alpha_1 = \alpha_2 = 1$?

Mom. equation $\sum F_s = \dot{m} V_2 - \dot{m} V_1$

neglecting shear stress, we lose that



$$p_1A_2 - p_2A_2 - \gamma A_2 L \sin\theta =$$

$$= \underbrace{\rho A_2 V_2}_{\dot{m}} \cdot V_2 + \rho A_1 V_1 V_1 \qquad \dot{m} = \rho A V$$

However for continuity

$$A_1 V_1 = A_2 V_2$$

So $\quad p_1 A_2 - p_2 A_2 - \gamma A_2 \overbrace{L \sin\theta}^{z} = \rho A_2 V_2^2 - \rho A_1 V_1 V_1 \qquad$ divide by $A_2\gamma$

$$\text{✯✯)}\quad \frac{p_1}{\gamma} - \frac{p_2}{\gamma} - (z_2 - z_1) = \frac{V_2^2}{g} - \frac{A_1}{A_2}\frac{V_1^2}{g}$$

21 ③ Putting * and ** together we have

$$① \quad \frac{p_1}{\gamma} + \frac{V_1^2}{g}\frac{A_1}{A_2} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{g} + z_2 \quad \text{Mom. eq}$$

$$② \quad \frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_1^2}{2g} + z_2 + \boxed{h_L} \quad \text{En. eq}$$

$$\boxed{h_L = \frac{(V_1 - V_2)^2}{2g}}$$ sudden expansion

if $V_2 = 0$ → then we have the exit loss

$$h_{exit} = \frac{V_1^2}{2g}$$

DEMONSTR:

subtract ① - ②

CONTIN.

$$\frac{V_1}{g}\cdot\frac{V_1A_1}{A_2} = \frac{V_1}{g}\frac{V_2A_2}{A_2} = \frac{V_1V_2}{g}$$

$$\frac{V_1}{g}\cdot\frac{V_2A_2}{A_2} = \frac{V_1V_2}{g}$$ apply continuity

$$\frac{V_1^2}{g}\frac{A_1}{A_2} - \frac{V_1^2}{2g} = \frac{V_2^2}{2g} - h_L$$

$$-h_L = \frac{V_2^2}{2g} - \frac{V_1^2}{2g} + \frac{V_1V_2}{g} \; ; \quad h_L = \frac{V_1^2}{2g} + \frac{V_2^2}{2g} - \frac{2V_1V_2}{2g}$$

$$\boxed{h_L = \frac{(V_1 - V_2)^2}{2g}}$$