2A1

example

Sea level pressure $p = 101\ kN/m^2$

Temperature $T = 4°C$ $\qquad$? ρ

Properties: AIR → GAS at $101\ \underbrace{kN/m^2}_{kPa}$, $4°C$ → $R = 287\ \frac{J}{K\,Kg}$

$$p = \rho R T \longrightarrow \rho = \frac{p}{RT} = \frac{101 \cdot 10^3\ N/m^2}{287\ J/kg\,K \cdot (273+4)\ K}$$

$$= 1.27 \cdot \frac{N}{m^2} \cdot \frac{kg\ K}{J} \frac{1}{K}$$

Remember $[J] = [N \cdot m]$

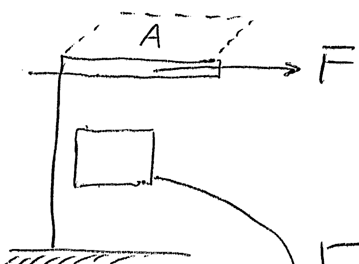
Energy = Force · lengthscale ;

$$\frac{\cancel{N}}{m^2} \cdot \frac{kg\ \cancel{K}}{\cancel{N} \cdot m} \frac{1}{\cancel{K}} = \frac{kg}{m^3}$$

$$\rho = 1.27\ \frac{kg}{m^3} \qquad \underline{\text{ok}} \qquad \left[\frac{M}{V}\right]$$

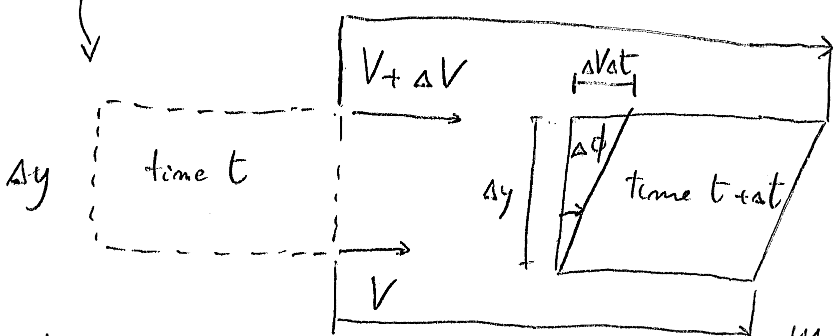
2B)

DAY 2 viscosity



$\tau = \frac{F}{A}$ shear stress $[Pa] = \left[\frac{N}{m^2}\right]$

$\boxed{\text{At time } t}$ $\boxed{\text{we define a square fluid element}}$ then, the fluid element start to move and to deform



in Δt the upper surface moves $(V + \Delta V) \cdot \Delta t$

The difference is $\Delta V \cdot \Delta t$

in the same Δt the lower surface moves $V \Delta t$

the strain rate is the rate at which Δφ change in time

The strain or deformation $\Delta \phi = \frac{\Delta V \cdot \Delta t}{\Delta y}$

$\alpha = \frac{l}{r}$

The rate of strain is $\frac{\Delta \phi}{\Delta t} = \frac{\Delta V}{\Delta y}$ → DIM. $\left[\frac{1}{s}\right] = \left[\frac{m/s}{m} = \frac{1}{s}\right]$ ok

as $\left.\begin{array}{l}\Delta t \to 0 \\ \Delta y \to 0\end{array}\right\}$ $\frac{d\phi}{dt} = \frac{dV}{dy}$ velocity gradient

The constitutive equation relates stress to deformation rate

In fluids $\tau = \mu \cdot \frac{dV}{dy}$

shear stress

rate of strain OR SHEAR RATE

DIM:

$\mu = \frac{\tau}{dV/dy}$

$[\mu] = \frac{N}{m^2} \cdot \frac{1}{\frac{m}{s} \cdot \frac{1}{m}} = \frac{N}{m}$

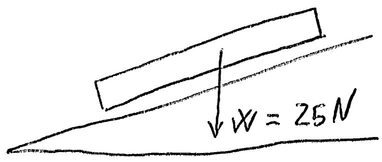
1 Poise → $0.1 \frac{Ns}{m^2}$

usually water $\mu = 10^{-3} \frac{Ns}{m^2}$ = 1 centipoise $= 10^{-2}$ Poise

2C

Example 2.3

Board sliding on an inclined plane $\theta = 20°$ with $V = 2\ cm/s$

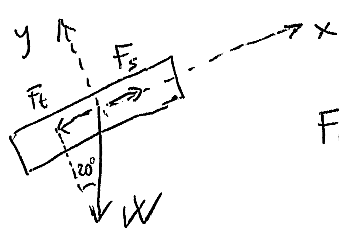


The board is seperated by a thin layer of oil

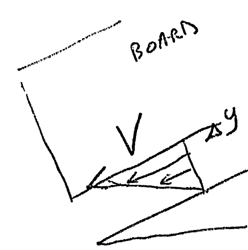
$\mu_{OIL} = 0.05 \frac{Ns}{m^2}$

? thickness of the oil layer

Forces acting on the board



$F_t = W \sin\theta$



CONSTANT velocity $\rightarrow \Sigma F = 0$

$F_t = F_s = \tau \cdot A$

ASSUME LINEAR velocity distrib.

but $\tau = \mu \frac{dV}{dy} = \frac{\Delta V}{\Delta y}$

$\Delta V = V - 0$

Δy = unknown ?

$W \sin\theta = \mu \frac{\Delta V}{\Delta y}$

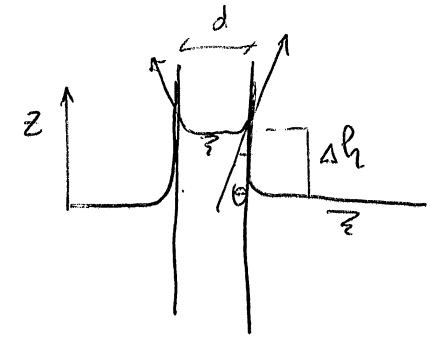
$V = \Delta V$ unit Area

Solve for $\Delta y = \frac{\mu \Delta V}{W \sin\theta} = \frac{0.05 \frac{Ns}{m^2} \cdot 0.02 \frac{m}{s} \cdot 1 m^2}{25 N \cdot \sin 20°}$

$\Delta y = 0.000117\ m = 0.117\ mm$

$\sim 100\ \mu m$

2E)

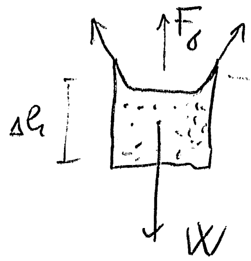


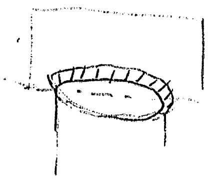
Given $T = 20°$; $d_{TUBE} = 1.6\,mm$

$\gamma_{water-AIR} = 0.073\ N/m$: $\gamma = 9790\ \frac{N}{m^3}$

Force balance

$$F_\gamma - W = 0$$





F_γ

the surface tension acts along the whole boundary (circumference) πd

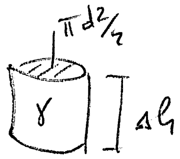
$$\underbrace{\gamma = \frac{F}{L} \rightarrow F = \gamma . L = \gamma \pi d}_{\text{we have to take the vertical component}}$$

$$F = \gamma \pi d \cdot \cos\theta$$

contact angle (very small) water and air

let us approx $\cos\theta \sim 1$

$$W = -\overbrace{\gamma \Delta h \cdot \frac{\pi d^2}{4}}^{\rho g \cdot V}$$



Equil.

$$\gamma \pi d - \gamma \Delta h \frac{\pi d^2}{4} = 0$$

$$\Delta h = \frac{4\gamma}{\gamma d} = \frac{4 \cdot 0.073}{9790 \cdot 1.6 \cdot 10^{-3}} = 18.6\,mm$$

Non negligible!

$d = 1.6\ mm$

$\Delta h = 18.6\ mm$!

$\boxed{3C}$ PRESSURE VARIAT.
in the atmosphere:

IDEAL GAS LAW

$$\rho = \frac{p}{RT}$$ multiply by gravity

$RT \rightarrow$ T in kelvin.

$$\rho g = \frac{p g}{RT} \quad ; \quad \gamma = \frac{p g}{RT}$$

US STANDARD ATMOSPHERE gives average conditions at sea level:

$T_0 = 15°C$: ($59°F$)

$p = 101.33$ kPa ABS.

$\rho = 1.225$ Kg/m^3

$\alpha = 5.87$ K/km LAPSE RATE

K: degree kelvin

Hydr. diff. eq.

$$\frac{dp}{dz} = -\gamma$$

$$\frac{dp}{dz} = -\frac{p g}{RT}$$

but $T = T_0 - \alpha(z - z_0)$ TROPOSPHERE ~ UP TO 13.7 km.

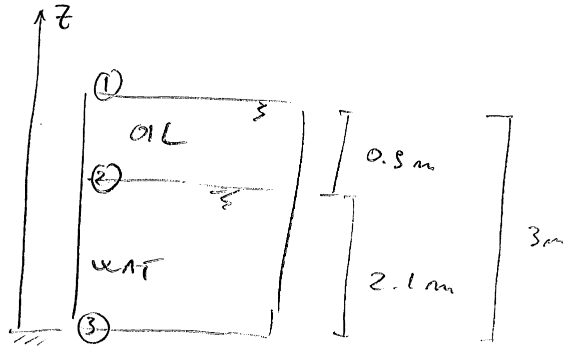
α: lapse rate

$$\frac{dp}{dz} = -\frac{p g}{R[T_0 - \alpha(z - z_0)]} \quad ; \quad \frac{dp}{p} = -\frac{g\, dz}{R(T_0 - \alpha(z - z_0))}$$

$$p = p_0 \left[\frac{T_0 - \alpha(z - z_0)}{T_0}\right]^{g/\alpha R}$$

3D

Pressure tank with 2 fluids.



OIL $\frac{\gamma_o}{\gamma_w} = 0.80$ $h_o = 0.9\,m$

specific gravity

LIGHTER

what is the gage pressure at the bottom of the tank?

Hydr. equat. OIL.

OIL 1–2

$$\frac{p_1}{\gamma_{OIL}} + z_1 = \frac{p_2}{\gamma_{OIL}} + z_2$$

$$\frac{0}{\gamma} + 3\,m = \underbrace{\frac{p_2}{0.8 \times 9810\,N/m^3}} + 2.1$$

GAGE pressure.

$$p_2 = 7.063\,kPa$$

OIL water – interface

$$p_2|_{OIL} = p_2|_{WATER} = 7.063\,kPa$$

WATER 2 – 3

Hydr. equat. water

$$\frac{p_2}{\gamma_{WAT}} + z_2 + \frac{p_3}{\gamma_{WAT}} + z_3$$

$$\frac{7.063 \times 10^3\,Pa}{9810\,N/m^3} + 2.1 = \frac{p_3}{9810} + 0 \longrightarrow p_3 = 27.7\,kPa \text{ gage}$$

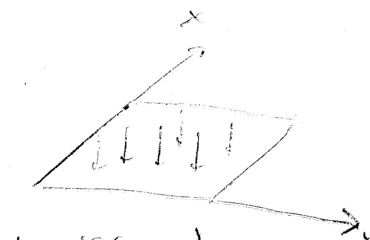
but oil is lighter so it is OK!

Note: we said that $1\,m^3$ WAT $\sim p \sim 10\,kPa$; 3m tank $\to p_3 \sim 30\,kPa$

4.1

Forces on plane surfaces

Pressure distribution $\longrightarrow$ $p = p(\bar{x})$ e.g.



$p = p(x, y)$

From the definition of pressure we can write

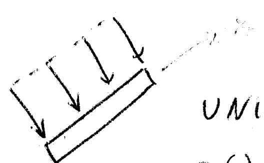
PREMESS. $p = \dfrac{dF_{NORMAL}}{dA}$ as $dA \to 0$

$$F = \int_A p \, dA = \bar{p} \cdot A$$

where $\bar{p}$ is the average pressure.

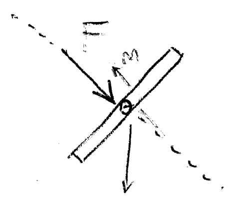
Where F is applied ?

Let us consider the simple case of a 1D panel



UNIF. PRESS. DISTR.

$p(x) = \tilde{p}$ CONST.



the point where F is applied is defined as center of pressure.

$F = \tilde{p} \cdot A \longrightarrow \boxed{\text{MAGNITUDE}}$

$\vec{F}$ unit vector $\perp A_{surface}$ $\boxed{\text{DIREC}}$

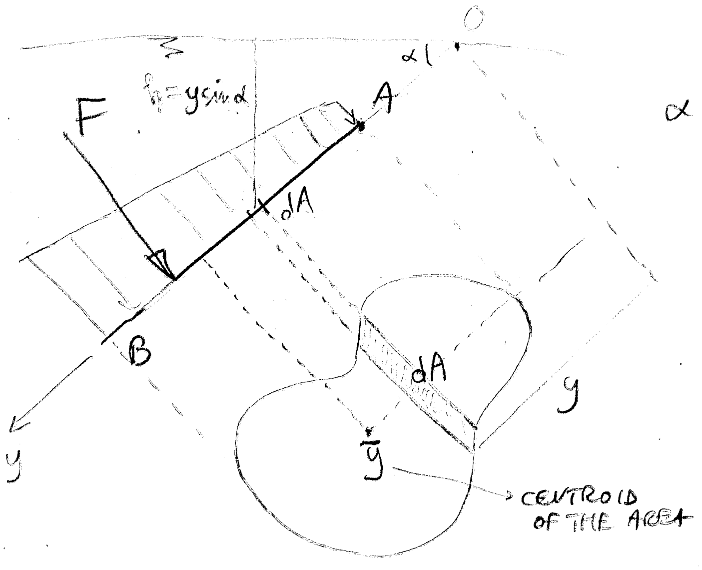
we use to define a surface with its normal vector n;

$F \,//\, n$

$|\hat{F} \cdot \hat{n}| = 1$

4B

Magnitude of the Resultant Hydrostatic Force



α : inclination of the panel
reference system on the slanted panel : $\vec{y}$ is inclined

On the surface portion dA (differential area) the pressure can be estimated as $p = \gamma \overbrace{y \sin\alpha}^{h}$

the differential force

$$dF = p\,dA = \gamma\, y \sin\alpha\, dA$$

The total force is the integral

$$F = \int_A p\,dA = \int_A \gamma\, y \sin\alpha\, dA$$

with same fluid $\gamma = \text{const.}$; panel → flat surface → $\alpha = \text{const.}$

$$F = \gamma \sin\alpha \underbrace{\int_A y\,dA}$$

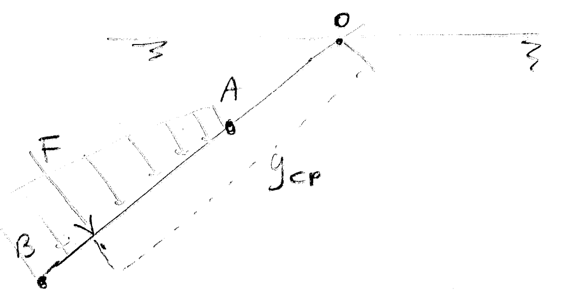
defined as the first moment of the AREA! $\bar{y}\cdot A = \int_A y\,dA$

CENTROID = center of gravity for homog. material density

$$F = \gamma\, \bar{y}\, A \sin\alpha = \underbrace{\gamma\, \bar{y} \sin\alpha}_{\text{= pressure at the centroid of the AREA}} \cdot A = \bar{p}\cdot A$$

4C1 Line of action of the resultant force.
(where it is applied?)
|
Center of pressure CP

Why is it important? → For the TORQUE BALANCE



Force · lever ARM (DISTANCE)
|
or Moment

Let us impose that the torque due to the resultant force F balance the torque due to the pressure distribution — otherwise we would create/destroy torque

In other words,

$$y_{cp} \cdot F = \int y \, dF$$

where $dF = p \, dA$

So
$$y_{cp} F = \int_A y \overbrace{p}^{p} \, dA = \int_A (\gamma y \sin\alpha) \, y \, dA = \int_A \gamma y^2 \sin\alpha \, dA =$$

$$= \gamma \sin\alpha \underbrace{\int_A y^2 \, dA}_{\text{Second Moment of an area}} = \gamma \sin\alpha \cdot I_0$$

Second Moment of an area
AREA MOMENT OF INERTIA

Mechanics:
measure the resistance of an object to change its ROTATION
↳ SKATING OR DIVERS get compact to reduce I and increase ROT. (ANGUL. vel.) → ANGM. MOMENT. $L = I \cdot \omega$

Since y_{cp} is defined with respect to O (ORIGIN),
the axis of rotation is in O !!! NOT STANDARD

4D

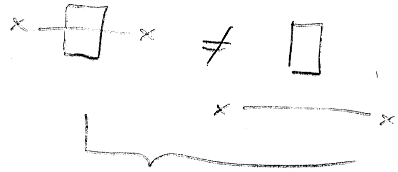
We can write

$$I_0 = \bar{I} + \bar{y}^2 \cdot A$$

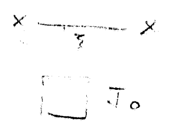
Remember $\bar{I}$ is the moment of inertia around the centroid of the AREA.

↓

the Mom. of inertia depends on the axis of ROTATION



we consider I_0 with respect to the surface, so



I_0

Parallel axis theorem state that!

$$y_{cp} \cdot F = \gamma \sin\alpha \left(\bar{I} + \bar{y}^2 \cdot A\right)$$

However we calculated $F = \gamma \bar{y} \sin\alpha \cdot A$

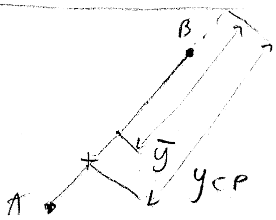
↓

$$y_{cp}\left(\cancel{\gamma} \bar{y} \cancel{\sin\alpha} A\right) = \cancel{\gamma} \cancel{\sin\alpha} \left(\bar{I} + \bar{y}^2 A\right)$$

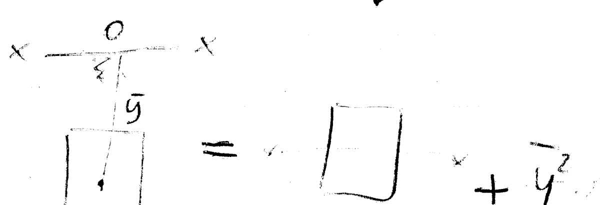
divide by $\bar{y}$

$$y_{cp} A = \bar{y} A + \frac{\bar{I}}{\bar{y}} \longrightarrow y_{cp} = \bar{y} + \underbrace{\frac{\bar{I}}{\bar{y} A}}_{>0}$$

Note $\bar{y}$ is the distance between the centroid and the water surface along the slanted panel!



y_{cp} is below $\bar{y}$



I_0 = Mom of inertia of A around O (surface)

$\bar{I}$

↓

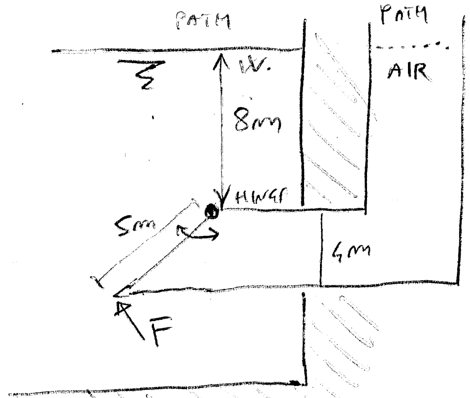
Mom of inertia of the surface around its own centroid

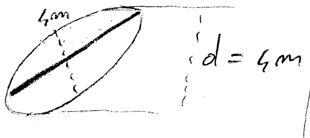
↓

TABLES !!!

7E Simple case.

Force to open an elliptical gate





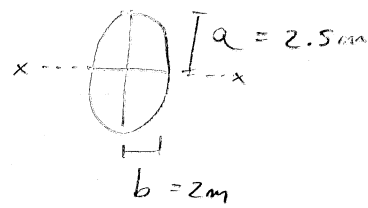


Fig A1 appendix

$A = \pi a b$

$I_{xx} = \dfrac{\pi a^3 b}{4}$

AROUND xx

a, b SEMIAXIS

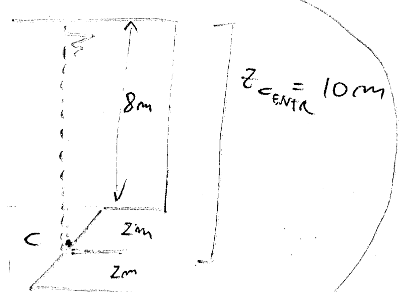
?) F able to open the gate

weight of the gate NEGLIGIBLE
FRICTION NEGLIGIBLE || assumptions

?) Let us calculate F

$\bar{p}$ = pressure at the centroid depth

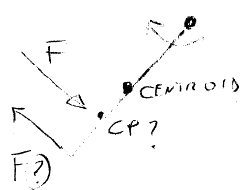
CENTROID DOES NOT DEPEND on pressure only on the area and shape.



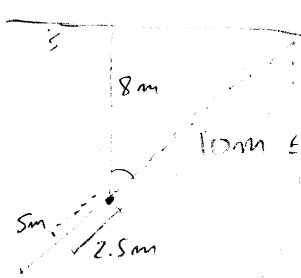
$\bar{p} = \gamma \cdot z_{CENTR.} = 9810 \frac{N}{m^3} \cdot 10 = 98.1 \text{ kPa}$

$A = \pi a b = \pi \cdot 2.5 \cdot 2 = 15.71 m^2$

$F = \bar{p} A = 98.1 \text{ kPa} \cdot 15.71 m^2 = 1.54 \boxed{MN} = 1.54 \cdot 10^6 N$ ||



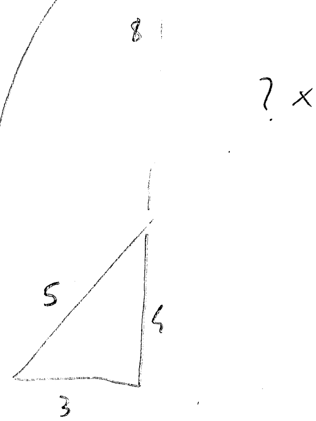
We need to find the center of pressure



$I = \dfrac{\pi a^3 b}{4} = \dfrac{\pi \cdot (2.5)^3 \cdot 2}{4} = 24.54 m^4$

4F) Now we can calculate

$$y_{cp} - \bar{y} = \frac{I}{\bar{y} \cdot A} = \frac{24.54 \text{ m}^4}{12.5 \times 15.71} = 0.125$$



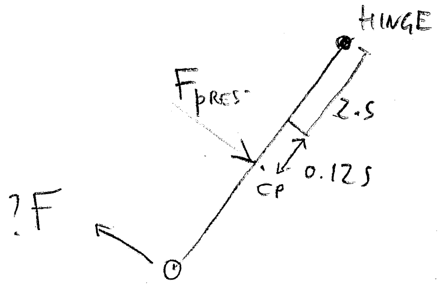
$$\frac{x}{5} = \frac{8}{4} \; ; \quad x = 10$$

$$\sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5$$

So the slanted centroid distance is

$10 + 2.5 = \underline{12.5}$

So the center of pressure is 0.125 below the CENTROID, along the slanted panel



Eventually let us estimate $F_?$

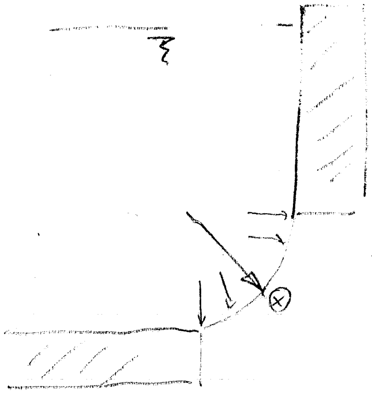
$$\Sigma M_{HINGE} = 0$$

$$F_? \times 5m = F_{PRES} \cdot 2.625 = 1.541 \, 10^6 \cdot 2.625$$

$$F_? = \frac{1.541 \cdot 10^6 \cdot 2.625}{5} = 808500 = \underline{808 \text{ kN}}$$

4G

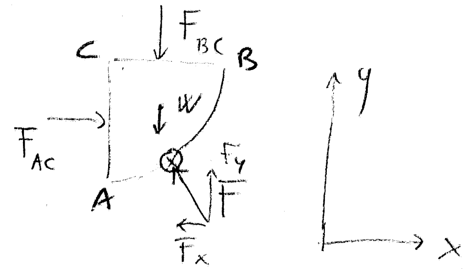
Forces on curved surfaces



Direct approach: integrate the pressure along the curved surface!

INDIRECT APPROACH

Free body DIAGRAM FBD



Let us consider the equilibrium of a portion of fluid

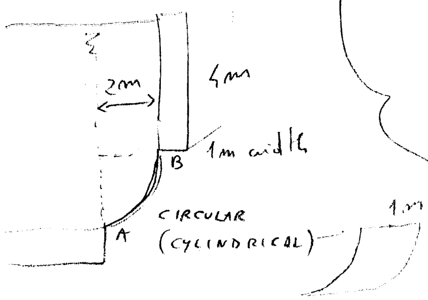
$\sum F_x = 0$

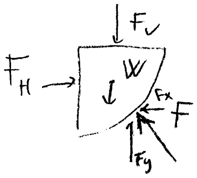
$F_x = F_{AC}$

$\sum F_y = 0$

$F_y = W + F_{cb}$

example



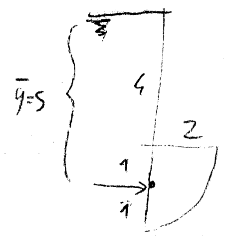


water 10°c ; $\gamma = 9810\ N/m^3$

Let us decompose F in a vert. F_y and horizontal F_x components

H. FORCE

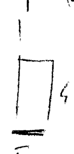
Eq (x) $\boxed{F_x} = F_H = p \cdot A = \gamma \bar{y} \cdot A = 9810 \cdot 5 \cdot \overset{A}{(2 \cdot 1)} = \underline{98.1\ kN}$



Eq (y)

$F_y = F_v + W$

$F_v = p \cdot A = \gamma \cdot 4 \cdot \overset{A}{(2 \cdot 1)} = 9810 \cdot 8 = 78.5\ k$



W weight $= \gamma V_{vol} = \gamma \cdot \frac{1}{4} \cdot \pi R^2 \cdot 1 =$

$= 9810 \cdot \frac{\pi \cdot 2^2}{4} = 30.8\ kN$

V. FORCE

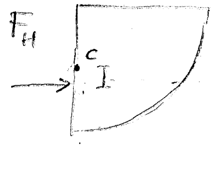
$\boxed{F_y} = F_v + W = 78.5 + 30.8 = 109.3\ kN$

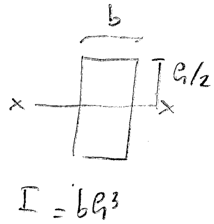
4H

Let us now determine the line of action of the force $\vec{F} = (F_x, F_y)$

Again let us consider the — HORIZ. force

$$y_{cp} = \bar{y} + \frac{I}{\bar{y}A}$$

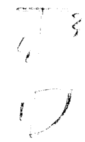




$I = \frac{bh^3}{12}$

$b = 1$
$h = 2$

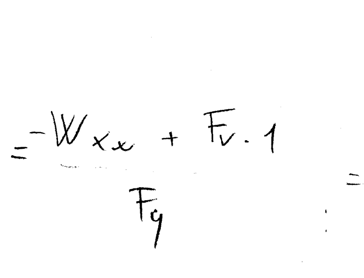
$\bar{y} = 4 + 1 = 5\,m$ → slanted panel is vertical



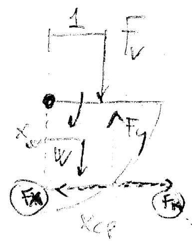
$$I = \frac{1 \cdot 2^3}{12}$$

$$y_{cp} = 5 + \frac{2^3/12}{5 \cdot (2 \cdot 1)}\ m = 5.067\,m$$

VERT FORCE



Moments around c



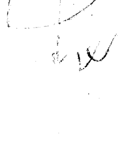
$F_v \cdot 1$

$x_w W$

$F_y \cdot x_{cp}$? BALANCE

Note: the moment between F_x and F_H is balanced.

$$x_{cp} \cdot F_y = 1 \cdot F_v + W \cdot x_w = 0$$



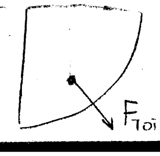
x_w = centroid of the arc

$$x'_{cp} = \frac{-W x_w + F_v \cdot 1}{F_y} =$$

Fig A_1

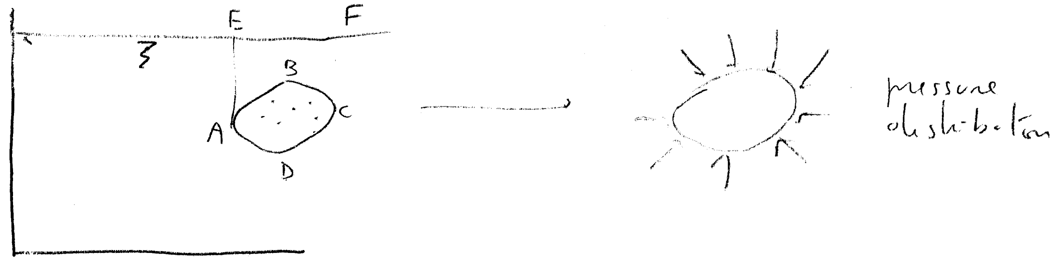
$\bar{x} = \frac{4}{3}\frac{r}{\pi}$, so $x_w = \frac{4}{3}\frac{2}{\pi} = 0.849\,m$

$$x'_{cp} = \frac{30.8 \cdot 0.849 + 78.5 \cdot 1}{109.3} = 0.957\,m$$



$$F_{TOT} = \sqrt{F_x^2 + F_y^2} = \sqrt{38.1^2 + 109.3^2} = 116\,kN$$

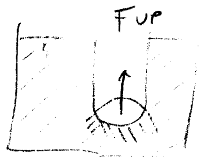
4I) Buoyant force equation



Let us consider
a body ABCD submerged
in a liquid of specific weight γ

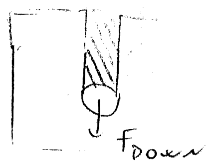
The pressure acting on the lower
portion of the body creates an UPWARD force = weight of liquid needed to fill the volume above the surface

$$F_{UP} = \gamma \left(V_{ABCD} + V_{ABCFE} \right)$$



Pressure on the top surface
create a downward force
= weight of the liquid
above the body

$$F_{DOWN} = \gamma V_{ABCFE}$$

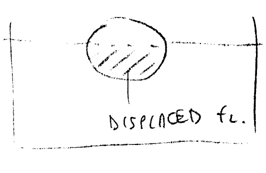


$$F_{BUOYANT} = F_{UP} - F_{DOWN} = \gamma V_{ABCD} = \gamma \underline{\underline{V_{BODY}}}$$

→ The net force or buoyant force = weight of the liquid that would be needed to occupy that volume

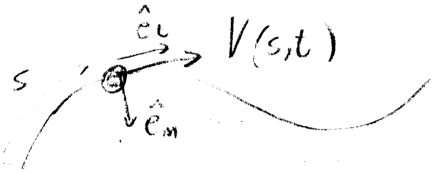
ARCHIMEDE'S principle

The buoyant force is equal
to the weight of the displaced
fluid



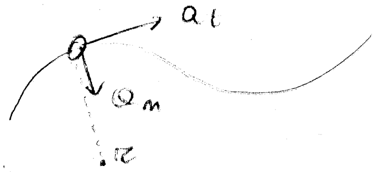
6A

Velocity and acceleration



velocity of a fluid particle along its path/trajectory

$V = V(s,t)$; $\vec{V} \parallel \vec{e}_t$; $\vec{V} = (V_t, 0)$ or $\bar{V} = V(s,t)\hat{e}_t$



Acceleration does have a tangential component and a normal component

$\bar{a} = (a_t, a_n)$

Any change of velocity is perceived as the result of an acceleration

where r is the local radius of curvature of the particle trajectory

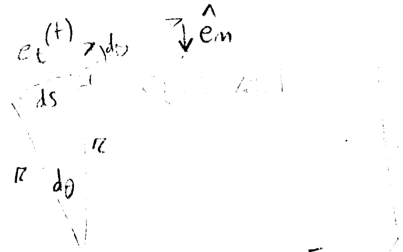
Given $\bar{V} = V(s,t)\,\hat{e}_t$

we can define acceleration $\bar{a} = \dfrac{d\bar{V}}{dt} = \dfrac{dV}{dt}\hat{e}_t + V\dfrac{d\hat{e}_t}{dt}$

while the magnitude of the vector cannot change in time, its direction does change.

note $s = s(t)$

$$\frac{dV(s,t)}{dt} = \frac{\partial V}{\partial s}\cdot\underbrace{\frac{ds}{dt}}_{\text{that is actually } V} + \frac{\partial V}{\partial t}\cdot\frac{dt}{dt} = \frac{\partial V}{\partial s}\cdot V + \frac{\partial V}{\partial t}$$



$$\frac{de_t}{dt} = \frac{d\theta}{dt} = \frac{d\theta}{ds}\cdot\frac{ds}{dt}$$

remember $d\theta = \dfrac{ds}{r}$, $\dfrac{1}{r}$, v

$$V\frac{d\hat{e}}{dt} = V\,\frac{V\hat{e}_n}{r}$$

$$\bar{a} = \left(V\frac{\partial V}{\partial s} + \frac{\partial V}{\partial t}\right)\hat{e}_t + \left(\frac{V^2}{r}\right)\hat{e}_n$$

6B

$$\bar{a} = \left(V \frac{\partial V}{\partial s} + \frac{\partial V}{\partial t}\right)\vec{e}_t + \left(\frac{V^2}{r}\right)\hat{e}_n = \frac{dV}{dt} = \bar{a}$$

By definition

VARIATION of the velocity with time at a point (FIXED) on the path line

= centripetal acceleration (strictly speaking it is a convective acceleration)

$\frac{\partial V}{\partial t} \rightarrow$ LOCAL ACCELERATION

in steady flow the local accel = 0

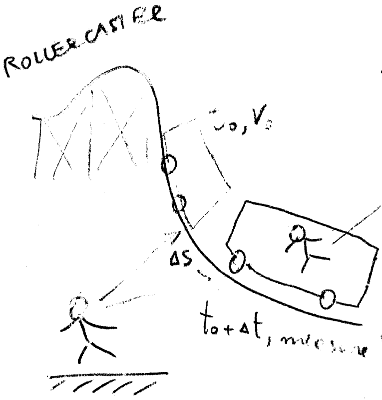
change of velocity DIRECTION along the path as opposed to a change in velocity magnitude along a path.

$V \frac{\partial V}{\partial s}$ is the variation of velocity along the pathline due to the transport of a SPATIAL VARIAT. of velocity (or due to the PARTICLE entering in a different flow region)

$V \frac{\partial V}{\partial s}$ is the CONVECTIVE ACCELERATION

in UNIFORM/homogeneous flow $V\frac{\partial V}{\partial s} = 0$

SKETCH



a passenger measures acceleration with an accelerometer; it read $\bar{a} = \frac{dV}{dt}$

In this case — note that $\Delta S = V\Delta t$ (first approx), so $V\frac{\Delta V}{\Delta S} = V\frac{\Delta V}{V\Delta t} = \frac{\Delta V}{\Delta t}$

$\frac{dV}{dt} \approx V\frac{\Delta V}{\Delta S}$ in the TANGENTIAL DIRECTION $\hat{e}_t$

$$V\frac{\partial V}{\partial s} \approx V\frac{\Delta V}{\Delta S}$$

But there is also a $\hat{e}_n$ component given by $\frac{V^2}{r}$

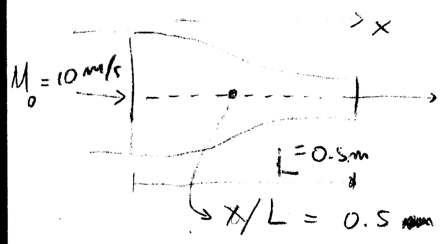
unless the track is locally linear $\rightarrow r \rightarrow \infty$ and $\frac{V^2}{r} = 0$

Note
the track/particle path is STATIONARY / STEADY $\Big\}\ \frac{\partial V}{\partial t} = 0$

Centripetal acceleration is estimated as $\frac{V^2}{r}$

pp. 83 Book is WRONG

6c] Example:



$\rightarrow u_1 = 20$ m/s , with GIVEN $u(x) = \dfrac{u_0}{1 - 0.5\dfrac{x}{L}}$

? acceleration at the nozzle midpoint $\rightarrow \frac{x}{L} = 0.5$

SOLUT.

1) pick the centerline path

2) calculate $u\dfrac{\partial u}{\partial x} = \overbrace{\dfrac{u_0}{(1-0.5\,x/L)^2}\cdot\left(-\dfrac{0.5}{L}\right)}^{\frac{\partial u}{\partial x}}\cdot\overbrace{\dfrac{u_0}{1-0.5\dfrac{x}{L}}}^{u} =$

$\quad\rightarrow = 0.5\,\dfrac{u_0^2}{L}\cdot\dfrac{1}{(1-0.5\,x/L)^3}$

for $\dfrac{x}{L} = 0.5 \quad u\dfrac{\partial u}{\partial x} = 0.5\cdot\dfrac{10^2}{0.5}\times\dfrac{1}{(1-0.5\cdot 0.5)^3} = 237\ \dfrac{m}{s^2}$

$$\bar{a} = \left(u\frac{\partial u}{\partial x} + \cancel{\frac{\partial u}{\partial t}}\right)\hat{e}_t + \cancel{\frac{V^2}{2}}\,\hat{e}_n = 237\ m/s^2\ \hat{e}_t$$

STEADY CASE $\qquad$ STRAIGHT PATH

Note, we have a net acceleration even if we are in steady conditions

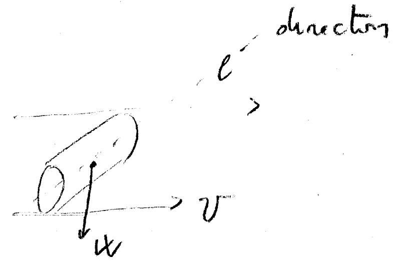
Euler eq.

↓

ALONG A PATHLINE

with direction l

Question: why there is no centripetal accel. in the Euler eq.



$$\left.\begin{matrix} \\ \end{matrix}\right\} \bar{F} m \cdot \bar{a} \text{ along } \underline{l}$$

↓

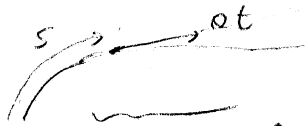
there is no $a_{centrip}$

$$-\frac{\partial}{\partial l}(p + \gamma z) = \rho a_l$$

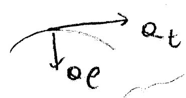
$l \parallel s$

we choose $s \to l$

$a_t \to a_l$



tangential component

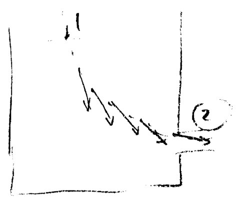


$a_l \perp a_t$

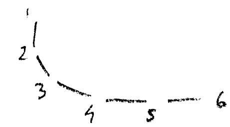
so $a_n \perp a_l$

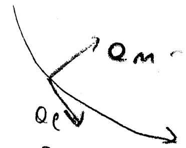
there is no component of centripetal accel. along $\underline{l}$

$\forall$ x, t along the path, I pick the tangent direction $\to$ $a_t = \dfrac{V\partial V}{\partial s} + \dfrac{\partial V}{\partial t}$ no centripetal.



$$\frac{d}{ds}\left(p + \gamma z + \rho\frac{V^2}{2}\right) = 0$$





$$\frac{\partial}{\partial n}(p + \gamma z) = \rho a_n \quad ?$$

$$\frac{\partial}{\partial l}(p + \gamma z) = \rho a_l$$

$$\frac{d}{dn}(p + \gamma z) = \underbrace{-\frac{\rho V^2}{R}}_{\text{centrip}}$$

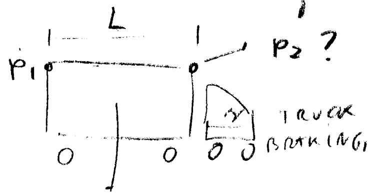
integrate, $\omega R = V$

$$\frac{d}{dn}\left(p + \gamma z - \frac{\omega^2 r^2}{2}\right) = 0$$

GEND OR 7BEG)

Example

Pressure in a decelerating tank of liquid

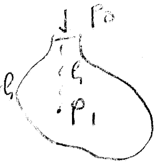


$\gamma = 42\ lbf/ft^3$

$a = -10\ ft/s^2$ (decel.)

$L = 20\ ft$

$p_1 = P_{ATM.}$ → Remember $p_1 = p_0 + \gamma l$



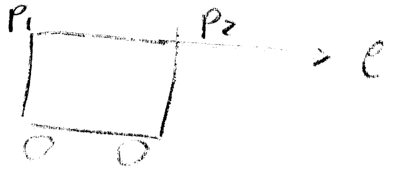
→ ATM. pressure is in 2 as well

p_2 ?

Euler eq.

$$-\frac{\partial}{\partial L}(p+\gamma z)=\rho a_L$$

Let us apply Euler eq. along l



$z = const$ → $\frac{dz}{dl} = 0$

$$\frac{dp}{dl} = -\rho a_l \quad \to \text{integrate} \quad (p_2 - p_1) = -\rho\int_{l_1}^{l_2} a_l \, dl$$

but $a_l = const$ so,

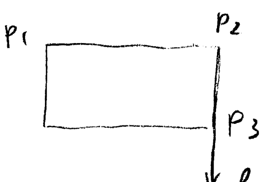
$$p_2 - p_1 = -\rho a_l \cdot \Delta l = -\frac{\gamma}{g} a_l L$$

$$p_2 = -\frac{42\ lbf/ft^3}{32.2\ ft/s^2}\cdot(-10\ ft/s^2)\cdot 20\ ft = 261\ psf_g$$

g — Pound per square feet — g gage

Question 2

Assuming TANK is 6ft height, what is the max pressure?



l Euler eq. along l,

$$\frac{d}{dz}(p+\gamma z) = -\rho a_z \ ; \ \text{but } a_z = 0$$

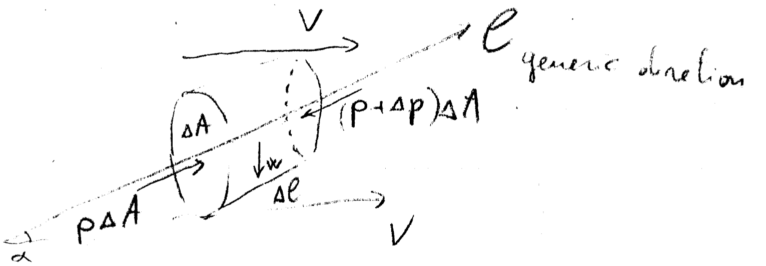
$\to p + \gamma z = const$ along l → $p_3 + \gamma z_3 = p_2 + \gamma z_2$; $p_3 = p_2 + \gamma(z_2 - z_3)$

$= 261 + 42 \times 6 = 513\ psf_g$

16D

Euler equation

Let us consider a cylindrical portion of a fluid in motion



Let us consider the general case in which the cylinder is accelerated along a general $\bar{\ell}$ direction $\neq \bar{V}$ (typical case of a curved trajectory)

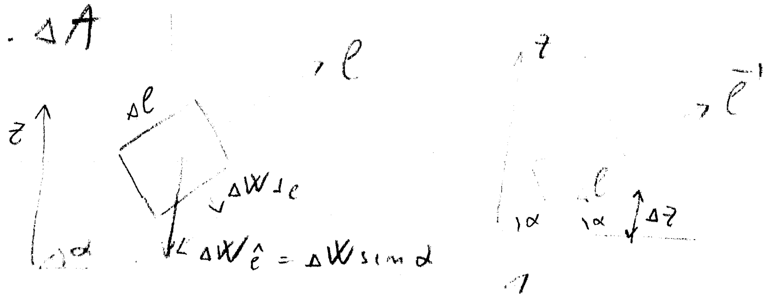
$$\sum F_\ell = m a_\ell \rightarrow \text{Total acceleration!}$$

$$F_{pressure} + F_{gravity} = m a_\ell$$

$$F_p = p\Delta A - (p+\Delta p)\Delta A = -\Delta p \cdot \Delta A$$

$$F_g = -\Delta W_\ell = -\Delta W \sin\alpha$$

$$F_g = -\Delta W \cdot \frac{\Delta z}{\Delta \ell}$$



Let us subst.

$$-\Delta p\, \Delta A - \underbrace{\gamma \Delta \ell \Delta A}_{\text{Weight } \Delta W = \gamma\, \Delta Volume = \gamma \Delta \ell \Delta A} \cdot \frac{\Delta z}{\Delta \ell} = \overbrace{\rho \Delta \ell \Delta A}^{m} \cdot a_\ell$$

Divide by $\Delta A \Delta \ell$

$$\frac{\Delta p}{\Delta \ell} - \gamma \frac{\Delta z}{\Delta \ell} = \rho a_\ell \xrightarrow[\text{as } \Delta \ell \to d\ell]{} \boxed{-\frac{\partial p}{\partial \ell} - \gamma \frac{\partial z}{\partial \ell} = \rho a_\ell}$$

6E

$$-\frac{\partial p}{\partial l} - \gamma \frac{\partial z}{\partial l} = \rho a_l$$

for an incompressible flow $\gamma = \text{const}$

$$\boxed{-\frac{\partial}{\partial l}\left(p + \gamma z\right) = \rho a_l}$$ along a generic $\hat{l}$ direction

Euler equation for motion in a fluid

1) Note we neglected the friction effect of the surrounding fluid

If a fluid particle tend to accelerate, viscous forces will oppose.

Euler equation works for an IDEAL FLUID (viscosity $= \emptyset$)

2) Note: if the fluid is STATIC, then $a_l = \emptyset$

$$\frac{\partial}{\partial l}(p + \gamma z) = 0 \rightarrow \underbrace{p + \gamma z}_{} = \text{CONST}$$

Hydrostatic equation 3.5

$$P_z = \underbrace{p + \gamma z}_{} = \text{CONST}$$ piezometric pressure

Note: this is TRUE for any direction $\vec{e}$

Correct: pressure is a scalar, independent of direction!

3)

$$\rho a_l = -\frac{\partial}{\partial l}(p + \gamma z)$$

minus.

the flow tend to accelerate to move from a high pressure to a low pressure

HIGH p_1 Δl p_2 LOW

$\longrightarrow l$

$$\Delta l > 0 \Big\} -\frac{\partial p}{\partial l} > 0 \Big\} \frac{\partial p}{\partial l} < 0 \; ; \; p_2 - p_1 < 0 \Big\} \; p_2 < p_1$$

Euler eq. incompressible inviscid fluids

$\boxed{\text{oF}}$

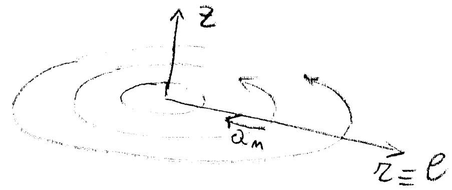
Pressure distribution in rotating flows

Meteorological analogy
high pressure → nice weather
low pressure → Hurricane bad weather

Euler: $-\dfrac{\partial}{\partial e}(p+\gamma z)=\rho a_e$

BOUNDARY MUST ROTATE e.g. a cylindrical container with a fluid inside

Let us consider a solid body ROTATION; steady flow $V=V(r)$
$r \perp$ streamlines



if the rotation velocity is constant,

then $\hat{a}=a\hat{e}_n$ ONLY normal/centripetal component

$$-\frac{d}{dr}(p+\gamma z)=\rho a_r=-\rho\frac{V^2}{r}$$

MINUS → accel is centripetal (TOWARDS THE CENTER) while $\vec{e}\equiv\vec{r}$ is directed OUTWARD

$$\frac{d}{dr}(p+\gamma z)=\rho\frac{V^2}{r}$$

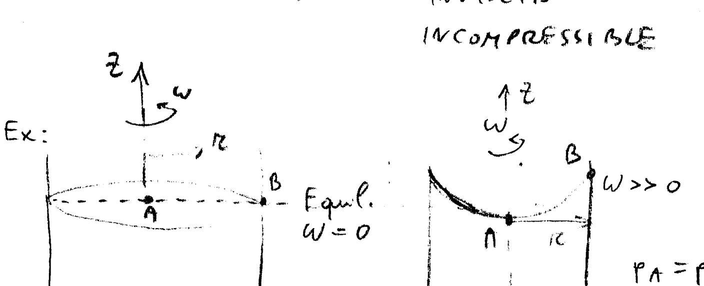
but $V=\omega r$ for SOLID BODY ROTATION

$$\frac{d}{dr}(p+\gamma z)=\rho r\omega^2 \rightarrow \text{integrate} \quad p+\gamma z-\rho\omega^2\frac{r^2}{2}=\text{CONST}$$

Remember: Assumptions: SOLID BODY ROTATION, INVISCID, INCOMPRESSIBLE

$$\boxed{\frac{p}{\gamma}+z-\frac{\omega^2 r^2}{2g}=C}$$

Ex:



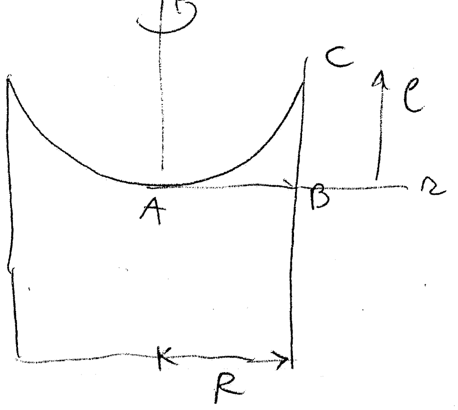
WRONG!

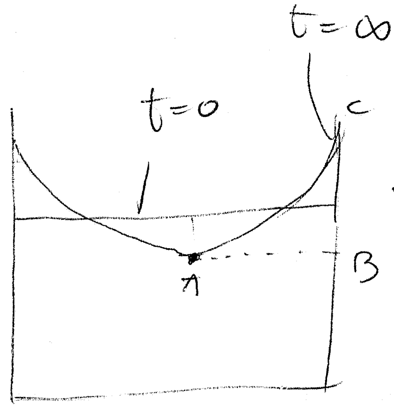
$p_A=p_B=\text{ATM.}$
$\omega_A^2=\omega_B^2$
$r_A=0$
$r_B=R$

$$z_A=-\frac{\omega^2R^2}{2g}+z_B\ ;\quad z_B-z_A=\frac{\omega^2R^2}{2g}$$

Note that in general $z=f(r^2)$; PARABOLIC PROFILE

6(G)





strictly speaking A → C

$\vec{AB}$ along r

$$\frac{p_A}{\gamma} + \cancel{z_A} - \cancel{\frac{\omega^2 r_A^2}{2g}} = \frac{p_B}{\gamma} + \cancel{z_B} - \frac{\omega^2 r_B^2}{2g}$$

$$z_A = z_B \; ; \; r_A = 0 \quad r_B^2 = R^2$$

$$p_B - \frac{\omega^2 R^2}{2g} \cdot \gamma = p_A$$

$$p_B = \frac{\omega^2 R^2}{2g} \gamma + p_A$$

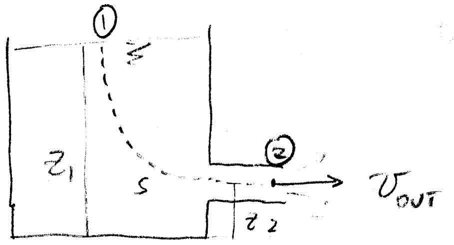
Now → BC along ℓ $\quad \frac{\partial (p + \gamma z)}{\partial \ell} = \rho a_\ell \qquad a_\ell = 0$

$$p_B + \gamma z_B = p_C + \gamma z_C$$

$$\frac{\omega^2 R^2}{2g} \gamma + \underbrace{p_A + \gamma z_B = p_C + \gamma z_C}_{p_A = p_C = ATM \; ; \; z_A = z_B}$$

$$z_C = z_A + \frac{\omega^2 R^2}{2g} \cdot \gamma$$ CORRECT.

7A]



Open TANK

given $\begin{cases} z_1 = 12 \text{ m} \\ z_2 = 2 \text{ m (centerline of the DRAIN TUBE)} \end{cases}$

? v_{OUT}

Approach

ASSUME ideal fluid
steady flow
velocity of the fluid on the water surface: negligible

Apply Bernoulli along a particular streamline ①---- (DRAIN) ②

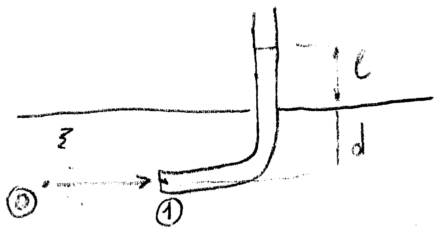
$$\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g} \quad \frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g} \quad \text{along } s$$

$p_1 = p_2 =$ ATM PRESS.

↓

$$z_1 - z_2 = \frac{V_2^2}{2g} \longrightarrow V_2 = \sqrt{2g(z_1 - z_2)} = \sqrt{2 \cdot 9.81 \cdot 10} = 14 \text{ m/s}$$

7B) Stagnation tube



Let us consider the streamline $0 \to 1$

apply Bernoulli

$$\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g} = p_0/\gamma + z_0 + \frac{V_0^2}{2g}$$

$z_1 = z_0$ HORIZ.

$V_1 = 0$ → STAGNATION POINT

$$\frac{p_1}{\gamma} = \frac{p_0}{\gamma} + \frac{V_0^2}{2g} \longrightarrow p_1 - p_0 = \frac{V_0^2}{2}\rho \longrightarrow \boxed{V_0^2 = \frac{2}{\rho}(p_1 - p_0)}$$

Since ① has $V_1 = 0$ we can apply HYDROST. EQUIL.

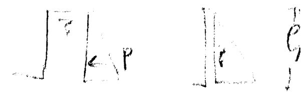
$$\left.\begin{aligned} p_1 &= \gamma(d+\ell) \\ p_0 &= \gamma d \end{aligned}\right\} \quad p_1 - p_0 = \overbrace{\gamma d + \gamma\ell}^{p_1} - \overbrace{\gamma d}^{p_0} = \gamma\ell$$

$$V_0^2 = \frac{2}{\rho}\cdot\gamma\ell = \frac{2}{\rho}\rho g\ell = 2g\ell$$

$$V_0 = \sqrt{2g\cdot\ell}$$

?) What if I use a larger pitot ?

it does not matter



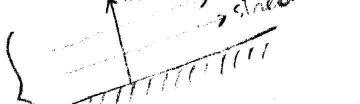
$F = \gamma V = \gamma h A$

$p = \frac{F}{A} = \frac{\gamma h A}{A} = \gamma h$ ok

Attention !

⓪ → fluid is moving,

why can I write $p_0 = \gamma d$? → Let us consider this case : $\vec{n} \perp$ streamline

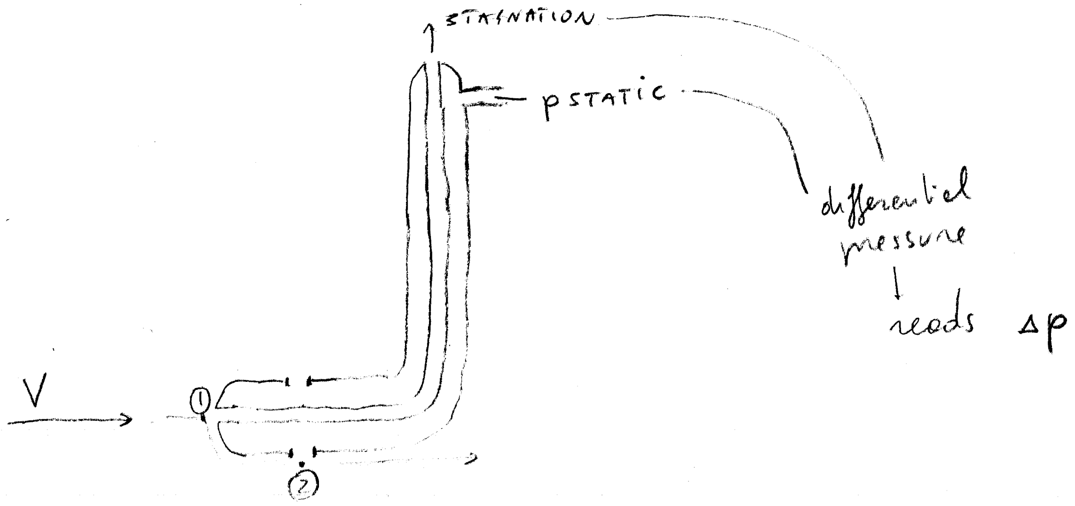


pressure across streamlines in parallel rectilinear flow has hydrostatic DISTRIBUT. ←

Euler eq along $\vec{n}$

$$\underbrace{\frac{\partial}{\partial n}(p + \gamma z)}_{p + \gamma n = \text{CONST.}} = \rho a_n \quad \text{but } \vec{n} \perp \vec{v} \text{ so } a_n = 0$$

7c)



Let us consider point 1 and 2 and apply Bernoulli

$$p_1 + \gamma z_1 + \frac{\rho V_1^2}{2} = p_2 + \gamma z_2 + \rho \frac{V_2^2}{2}$$

$$V_1 = 0$$

$$V_2 = \sqrt{\frac{2}{\rho}\left(\underset{p_1+\gamma z_1}{p z_1} - \underset{p_2+\gamma z_2}{p z_2}\right)}$$ piez. pressure

where pz is the piezometric pressure $= p + \gamma z$

$$V = \sqrt{\frac{2}{\rho}\Delta p_z}$$

since the instrument is small, and usually the pitot tube is used in AIR flows (γ is small) then

$$V = \sqrt{\frac{2}{\rho}\Delta p}$$

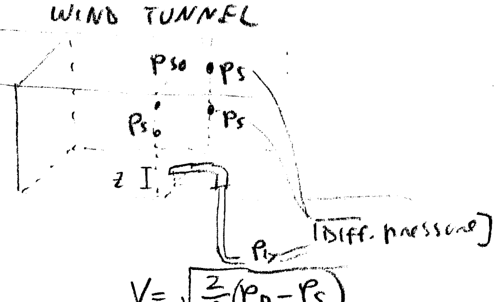
Differential Pressure transducer

measure deformation of a membrane

Note: if the pipe is pressurized,

$$P_{STAGNATION} = P_{velocity} + P_{fluid}$$

So the STAGNATION tube cannot work, but the PITOT yes

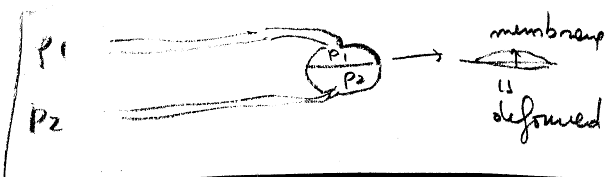


$$V = \sqrt{\frac{2}{\rho}(P_D - P_s)}$$

It is important that DYNAM. & STATIC pressure are measured closely

SAME CROSS SECTION

but you do not want ~~acceleration~~ RECIRCULATION



11A | Review

Fluid property

1) $\rho = \lim_{\Delta m \to 0} \frac{\Delta m}{\Delta Vol} \sim \frac{m}{Vol}$ $[kg/m^3]$

2) $\rho \cdot g = \gamma$ specific weight $[\gamma] = [\rho] \cdot [g] = \frac{kg}{m^3} \cdot \frac{m}{s^2} = \frac{N}{m^3}$

$\gamma^{H_2O}_{20^\circ c} = 9790 \ N/m^3$ $\gamma_{AIR \ 20^\circ c} = 11.8 \ \frac{N}{m^3}$

$\gamma^{H_2O}_{50^\circ F} = 62.4 \ \frac{lbf}{f^3}$

3) ideal gas $\rho = \frac{P}{RT}$ (KELVIN) with R GAS CONSTANT AIR at $15^\circ c$, $p = 1$ ATM $= 287 \ \frac{J}{Kg \ ^\circ k}$

Viscosity

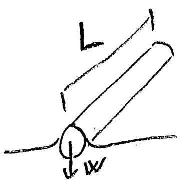
4) $\mu = \frac{\tau}{dV/dy}$ where τ (shear stress) $= \frac{F_{//AREA//MOTION}}{AREA}$ $[\tau] = \frac{N}{m^2} = Pa$

$[\mu] = \frac{N \cdot s}{m^2}$

5) ν Kinematic viscosity $\nu = \frac{\mu}{\rho}$, $[\nu] = \frac{N \cdot s}{m^2 \ kg/m^3} = \frac{m^2}{s}$

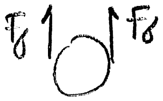
6) Surface Tension $\sigma_{H_2O \ AIR} = 0.073 \ \frac{N}{m}$ ROOM TEMPER.

$F = \sigma \cdot L$ → length over which surface tension acts



FOR A CYLINDER

$2 \cdot F_\sigma = W$ with $F_\sigma = \sigma \cdot L$

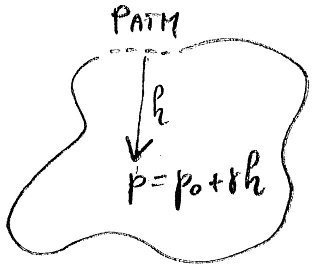


$\boxed{11B}$ Pressure

$$p = \lim_{\Delta A \to 0} \frac{\Delta F}{\Delta A} \sim \frac{F}{A} \qquad [p] = \frac{N}{m^2} = Pa$$

$$P_{ABS} = P_{ATM} + \underbrace{P_{GAGE}}$$

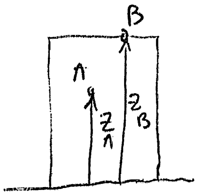
actual pressure measured by a device



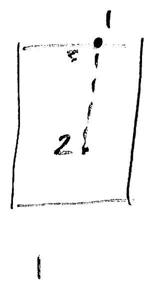
HYDR. EQ.

$$\left.\Sigma F = 0\right\} \; p + \gamma z = \text{CONST}$$

in a fluid with CONSTANT DENSITY (SAME FLUID)

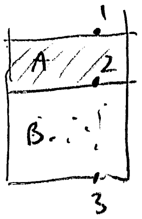


piezom. head $h = \frac{p}{\gamma} + z = \text{CONST}$



$$\frac{p_1}{\gamma} + z_1 = \frac{p_2}{\gamma} + z_2$$

If you have a ≠ fluid



$$\frac{p_1}{\gamma_A} + z_1 = \frac{p_2}{\gamma_B} + z_2 : \Big\} \; h \text{ CONST } A$$

$$p_2|_A = p_2|_B \Big\} \; p \text{ CONST at the interface}$$

$$\frac{p_2}{\gamma_B} + z_2 = \frac{p_3}{\gamma_B} + z_3 \Big\} \; h \text{ const } B$$

FORCES ACTING ON A SURFACE

$$F = \int_A p\,dA \; ; \qquad \text{if} \quad p = \gamma y' \quad \text{HYDR. PRESS. DISTR. (VERTICAL PANEL)}$$

VERTICAL

$$\text{then} \quad F = \gamma \int_A y\,dA = \gamma \bar{y}$$ → centroid of the area.

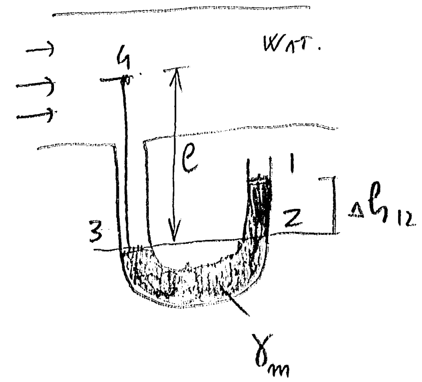
where the force is applied? (center of pressure)

$$y_{cp} - \bar{y} = \frac{I}{\bar{y} A}$$

moment of inertia around an axis that passes through $\bar{y}$

TABLES

Exercises
on the MANOMETERS



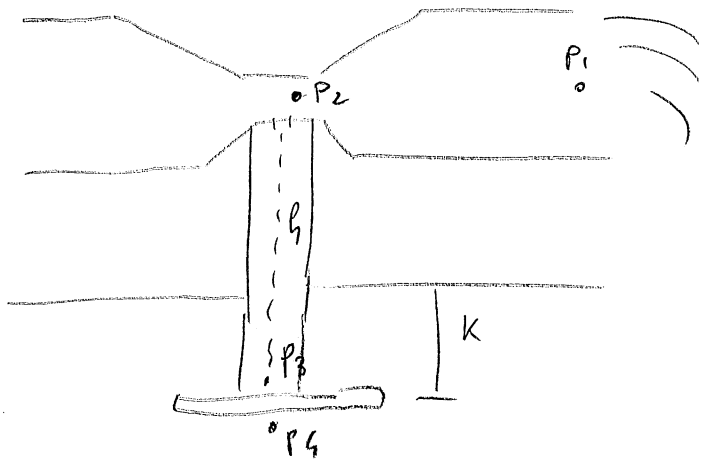
$$p_2 = P_{ATM} + P_1 = \emptyset + \gamma_m \Delta h_{12}$$

MANOMETER working fluid

$$p_3 = p_4 + \gamma_w \cdot \ell$$

$$p_2 = p_3$$

$$\gamma_m \Delta h_{12} = p_4 + \gamma_w \cdot \ell$$



$P_1, P_2 \to$ Bernoulli

$V_2 \gg V_1$ $\quad p_2$ low

low low $p_2 = p_{vap}$

$$p_3 = p_2 + \gamma h$$

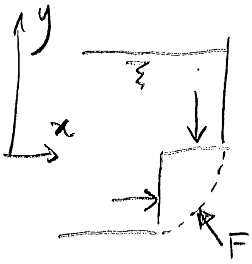
but $p_4 = P_{ATM} = \gamma K$

$$F_{LIFT} = A \cdot (p_4 - p_3) = \ldots$$

11C] Forces on curved surfaces

a few tricks

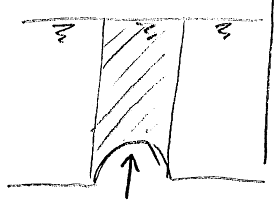
- free body diagram



F_y DECOMPOSE THE PROBLEM
F_x in the 2 DIRECTIONS x, y

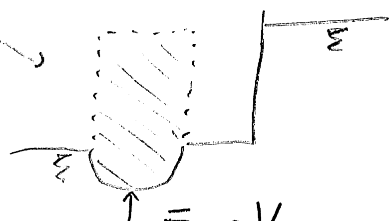
- let us consider the mass of fluid acting on the surface

or what is necessary to keep the fluid in EQUILIBRIUM c.f.

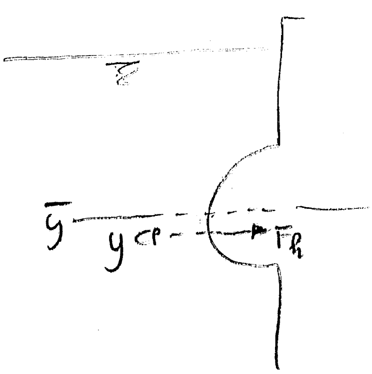


$F = \rho V_{\boxtimes}$

the mass of fluid that would be necessary to have STATIC CONDIT.



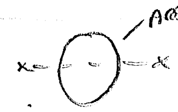
$F = \rho V_{\boxtimes}$



Dome → area = πr^2

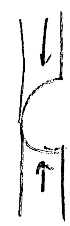
$F_h = \gamma \bar{y} \cdot \pi r^2$

$y_{cp} - \bar{y} = \dfrac{I}{\bar{y} A}$ → with $I = \dfrac{\pi r^4}{4}$ circle



it's not  this

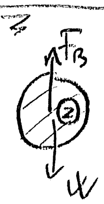
$F_V = ?$



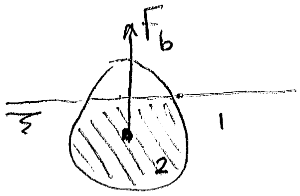
≡ EQUIV.

$= \rho \cdot \underbrace{\frac{1}{2} \cdot \underbrace{\frac{4}{3}\pi r^3}_{\text{VOL DOME}}}_{\text{VOL. SEMI DOME}}$

11 D Buoyant force



① $F_B = \rho_1 g V_{OL}$ → replacing that volume with fluid ① would guarantee HYDR. STATIC



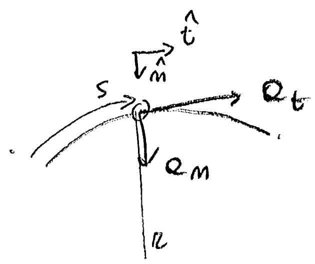
PARTIALLY SUBM.

$F_b = \rho_1 g V_{OL}$

Fluid MOTION $V = V(s,t)$

$\frac{\partial V}{\partial s} = 0$ UNIFORM

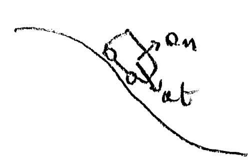
$\frac{\partial V}{\partial t} = 0$ STEADY



$$\bar{a} = \underbrace{\frac{d\vec{V}}{dt} \text{ along a trajectory / pathline}}_{\text{LAGRANG. ACCEL.}} = \left(V\frac{\partial V}{\partial s} + \frac{\partial V}{\partial t} \right)\hat{t} + \frac{V^2}{R}\hat{n}$$

convective — eulerian local — centripetal

If we measure directly LAGRANG. accel., we should make sure that the accelerometer is properly oriented! ↗ we can measure a_t and a_n



along $\hat{t}$ INVISCID, INCOMPRESS. fluid Euler equation

for generic direction ℓ → ΣF_m along ℓ

$$\frac{\partial}{\partial \ell}(p + \gamma z) = \rho a_\ell$$

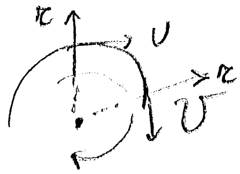
when $\ell \perp$ streamlines, $a_\ell = 0$

$p + \gamma z$ = constant along $n \perp$ streamlines

11E

Press. distrib. in ROTATING FLOWS

apply Euler in direct. $\perp$ velocity $\quad \frac{d}{dr}(p+\gamma z) = -\rho\frac{V^2}{r}$



$$a = -\frac{V^2}{r}$$

RIGID BODY ROTAT.

$V = \omega r$

$$p + \gamma z \boxed{-} \rho\frac{\omega^2 r^2}{2} = \text{CONST. along } r$$

$\perp$ streamlines.

note that these 2 equations are different ($\pm \frac{V^2}{2}\rho$)

so Bernoulli does not apply $\perp$ to / ACROSS streamlines

Bernoulli!

apply Euler eq. along a streamline in steady flow

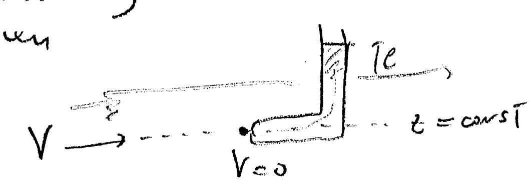
INVISCID, STEADY, INCOMPRESS.

$$p + \gamma z + \rho\frac{V^2}{2} = \text{CONST along streamline}$$

TANGENT.

$$\frac{p}{\gamma} + z \boxed{+} \frac{V^2}{2g} = \text{const}$$

important! apply Bernoulli along streamlines and compare points where boundary conditions are known



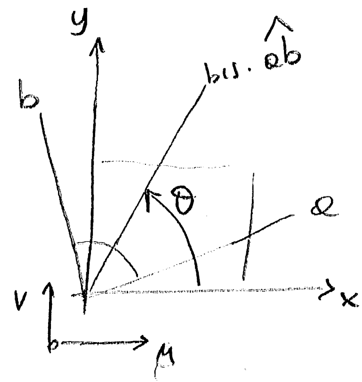
1F VORTICITY

$$\vec{\omega} = 2\vec{\Omega} = \left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right)\hat{i} + \left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right)\hat{j} + \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)\hat{k}$$

where $\vec{\Omega}$ is the ROTATIONAL velocity RATE

2D case

$$\Omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = \dot{\theta}$$



where $\dot{\theta} = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t}$

note

$$[\Omega_z] = \left[\frac{RAD}{s}\right]$$

IRROTATIONAL FLOWS $\rightarrow$ $\vec{\Omega} = 0$ in 2D $\rightarrow$ $\underbrace{\Omega_z = 0}_{\frac{\partial v}{\partial x} = \frac{\partial u}{\partial y}}$

Forced vortex

$V = \omega R$ ROTAT.

$\Omega_z = \omega$ RIGID BODY ROTAT.

Free vortex

$V = \frac{C}{r}$ IRROT.

$\Omega_z = 0$

if the flow is IRROT. we apply Euler eq. $\perp$ streamline

$$p + \gamma z + \rho\frac{V^2}{2} = \text{const}$$

but if we apply Euler // streamline $\rightarrow$ Bernoulli

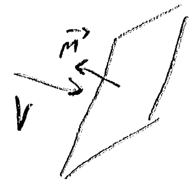
INCOMPRESS. STEADY, INVISCID, IRROT. $\rightarrow$ $\frac{p}{\gamma} + z + \frac{V^2}{2g} = \text{CONST}$ between any POINTS

OK // streaml. & $\perp$ streamline

11 G]   Rate of flow or volume flow rate
or discharge

$$Q = \int_A \vec{V} \cdot d\vec{A} \qquad \text{if} \quad \vec{V} \perp \vec{A} \; \Big\} \; Q = \int_A V\, dA$$

$$V \parallel \vec{n}$$



Note $[Q] = \frac{m^3}{s}$

if $V = \text{const}$ over A $\longrightarrow$ $Q = VA$

Given $Q \longrightarrow$ I can define

$$\underset{\text{mean velocity}}{\overline{V}} = \frac{Q}{A} \quad [m/s]$$

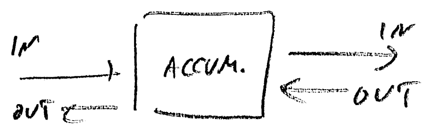
Similarly the mass flow rate

$$\dot{m} = \int_A \rho \vec{V} \cdot d\vec{A}$$

Definition of
CONTROL VOLUME
(fixed! in time)

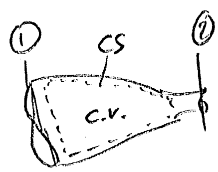
Continuity equation

$$\underbrace{\frac{d}{dt} \int_{C.V.} \rho \, dV_{ol}}_{\text{change in mass in the c.v.}} + \underbrace{\int_{C.S} \rho \vec{V} \cdot d\vec{A}}_{\text{net flux of mass through the c.s}} = 0$$



$$\frac{d}{dt} m_{c.v.} + \sum_{CS} \dot{m}_{OUT} - \sum_{C.S} \dot{m}_{IN} = 0$$

In pipes
if flow is steady:



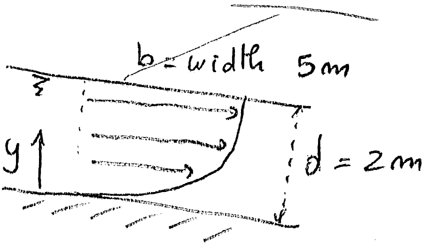
then $\dot{m}_1 = \dot{m}_2$

$Q_1 = Q_2$ $\Big\}$ in general $\sum_{C.S} Q_{in} = \sum_{C.S} Q_{OUT}$

$V_1 A_1 = V_2 A_2$

TO DO:
- Hydr. machines
- MANOMETRY

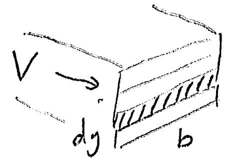
Example



water velocity in a channel

$$\frac{u(y)}{u_{MAX}} = \left(\frac{y}{d}\right)^{\frac{1}{2}}$$ DISTRIB. of velocity GIVEN ; $u_{MAX} = 3\,m/s$

2) Discharge



$$Q = \int_A V\,dA \quad \text{but} \quad dA = b\,dy = 5\,dy$$

$$Q = \int_0^d u(y)\,b\,dy = \int_0^d u_{MAX}\cdot\left(\frac{y}{d}\right)^{\frac{1}{2}}\cdot 5\,dy$$

$$= \frac{u_{MAX}\cdot 5}{d^{1/2}} \underbrace{\int_0^d y^{1/2}\,dy}$$

$$= \frac{5\,u_{MAX}}{d^{1/2}}\left[\frac{2}{3}y^{3/2}\right]_0^d = \frac{5\cdot\cancel{3}}{2^{1/2}}\left[\frac{2}{\cancel{3}}2^{3/2}\right] = 20\,\frac{m^3}{s}$$

What is the mean velocity $\bar{V} = \frac{Q}{A} = \frac{20\,m^3/s}{5\cdot 2\,m^2} = 2\,m/s$

What is the mass flow rate ? $\dot{m} = \rho Q = 1000\,\frac{kg}{m^3}\,20\,\frac{m^3}{s} = \frac{20\,t}{s}$

11A | Review

Fluid property

1) $\rho = \lim_{\Delta m \to 0} \frac{\Delta m}{\Delta Vol} \sim \frac{m}{Vol}$ $[kg/m^3]$

2) $\rho \cdot g = \gamma$ specific weight $[\gamma] = [\rho]\cdot[g] = \frac{kg}{m^3}\cdot\frac{m}{s^2} = \frac{N}{m^3}$

$\gamma^{H_2O}_{20°c} = 9790\ N/m^3$ $\gamma_{AIR\,20°c} = 11.8\ \frac{N}{m^3}$

$\gamma^{H_2O}_{50°F} = 62.4\ \frac{lbf}{f^3}$

3) ideal gas $\rho = \frac{p}{RT}$ (T KELVIN) with R GAS CONSTANT AIR at 15°c, $p = 1$ ATM $= 287\ \frac{J}{Kg\ °k}$

Viscosity

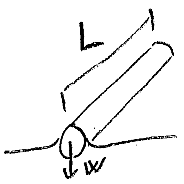
4) $\mu = \frac{\tau}{dV/dy}$ where τ (shear stress) $= \frac{F_{//AREA//MOTION}}{AREA}$ $[\tau] = \frac{N}{m^2} = Pa$

$[\mu] = \frac{N \cdot s}{m^2}$

5) ν Kinematic viscosity $\nu = \frac{\mu}{\rho}$, $[\nu] = \frac{N \cdot s}{m^2\ kg/m^3} = \frac{m^2}{s}$

6) Surface Tension $\sigma_{H_2O\ AIR} = 0.073\ \frac{N}{m}$ ROOM TEMPER.

$F = \sigma \cdot L$ → length over which surface tension acts



FOR A CYLINDER

$2 \cdot F_\sigma = W$ with $F_\sigma = \sigma \cdot L$ $F_\sigma \uparrow \bigcirc \uparrow F_\sigma$

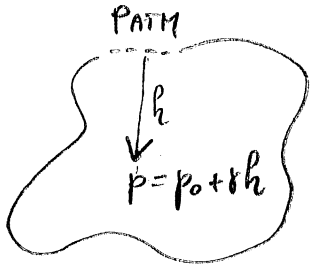
11B

Pressure

$$p = \lim_{\Delta A \to 0} \frac{\Delta F}{\Delta A} \sim \frac{F}{A} \qquad [p] = \frac{N}{m^2} = Pa$$

$P_{ABS} = P_{ATM} + P_{GAGE}$

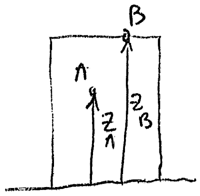
actual pressure measured by a device



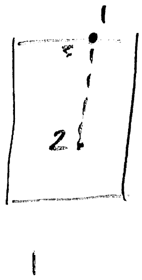
HYDR. EQ.

$\Sigma F = 0$ } $p + \gamma z = \text{CONST}$

in a fluid with CONSTANT DENSITY (SAME FLUID)

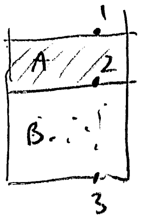


piezom. head $h = \dfrac{p}{\gamma} + z = \text{CONST}$



$$\frac{p_1}{\gamma} + z_1 = \frac{p_2}{\gamma} + z_2$$

If you have a ≠ fluid



$\dfrac{p_1}{\gamma_A} + z_1 = \dfrac{p_2}{\gamma_B} + z_2$: } h const A

$p_2|_A = p_2|_B$ } p const at the interface

$\dfrac{p_2}{\gamma_B} + z_2 = \dfrac{p_3}{\gamma_B} + z_3$ } h const B

Forces acting on a surface

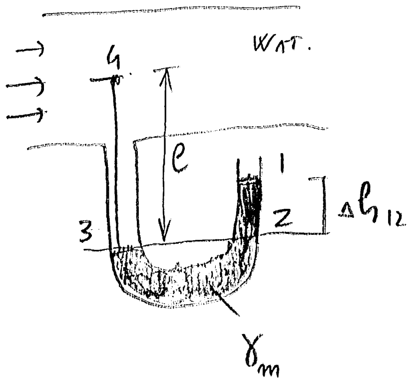
$F = \int_A p\, dA$; if $p = \gamma y$ (VERTICAL) HYDR. PRESS. DISTR. (VERTICAL PANEL)

then $F = \gamma \int_A y\, dA = \gamma \bar{y}$ → centroid of the area.

where the force is applied? (center of pressure)

$$y_{cp} - \bar{y} = \frac{\bar{I}}{\bar{y} A}$$ → moment of inertia around an axis that passes through $\bar{y}$ TABLES

Exercises
on the MANOMETERS



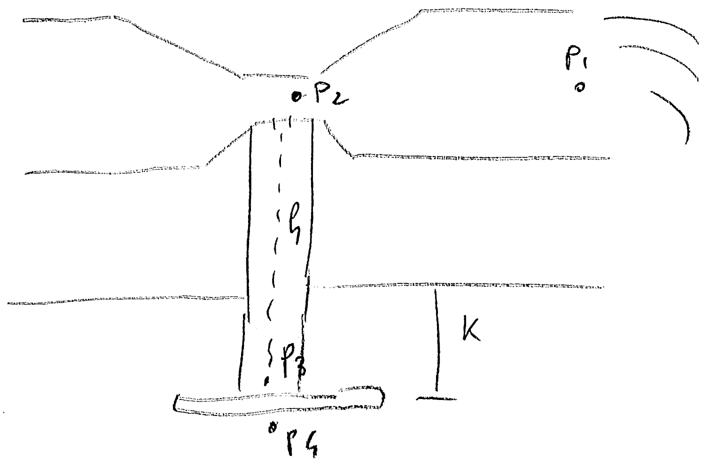
$$p_2 = p_{ATM} + p_1 = \emptyset + \gamma_m \, \Delta h_{12}$$

MANOMETER working fluid

$$p_3 = p_4 + \gamma_w \cdot l$$

$$p_2 = p_3$$

$$\gamma_m \, \Delta h_{12} = p_4 + \gamma_w \cdot l$$



$p_1, p_2 \rightarrow$ Bernoulli

$V_2 \gg V_1$ $\quad p_2$ low

low low $p_2 = p_{vap}$

$$p_3 = p_2 + \gamma h$$

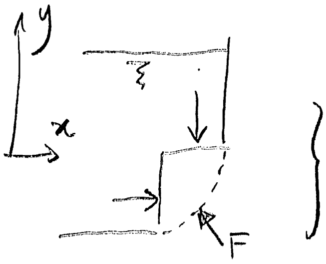
but $p_4 = p_{ATM} = \gamma K$

$$F_{LIFT} = A \cdot (p_4 - p_3) = \ldots$$

11C] Forces on curved surfaces

a few tricks

- free body diAGRAM

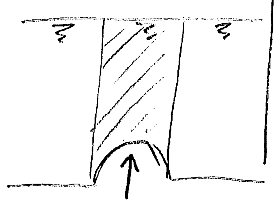


F_y DECOMPOSE THE PROBLEM

F_x in the 2 DIRECTIONS x, y

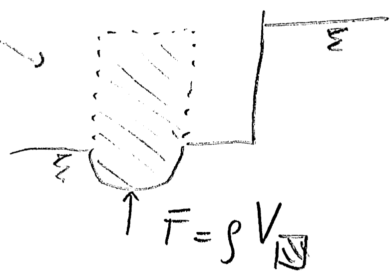
- let us consider the mass of fluid acting on the surface

or what is necessary to keep the fluid in EQUILIBRIUM c.f.

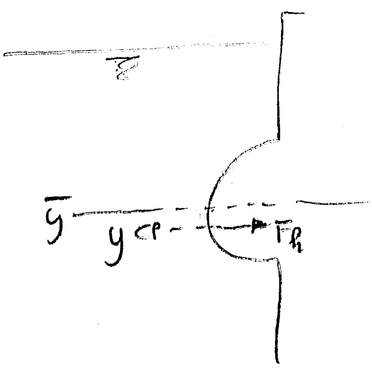


$F = \rho V$

the mass of fluid that would be necessary to have STATIC CONDIT.



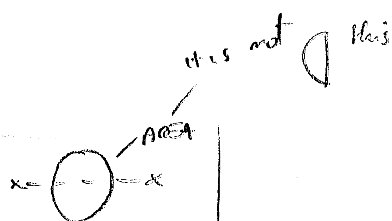
$F = \rho V$



Dome → area = πr^2

$F_R = \gamma \bar{y} \cdot \pi r^2$

$y_{cp} - \bar{y} = \dfrac{I}{\bar{y} A}$ → with $I = \dfrac{\pi r^4}{4}$ circle

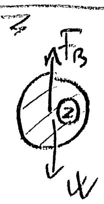


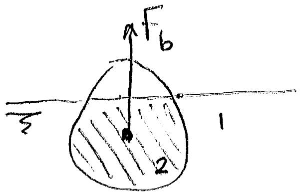
$F_V = ?$

≡ EQUIV.

$= \rho \cdot \underbrace{\frac{1}{2} \cdot \underbrace{\frac{4}{3} \pi r^3}_{\text{VOL DOME}}}^{\text{VOL. SEMI DOME}}$

11 D | Buoyant force

①  $F_B = \rho_1 g V_{OL}$ → replacing that volume with fluid ① would guarantee HYDR. STATIC



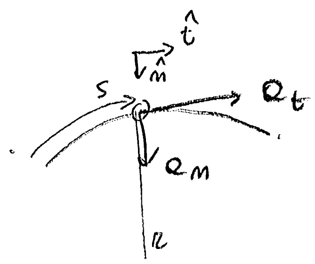
PARTIALLY SUBM.

$F_b = \rho_1 g V_{OL}$

Fluid MOTION $V = V(s,t)$

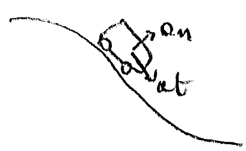
$\frac{\partial V}{\partial s} = 0$ UNIFORM

$\frac{\partial V}{\partial t} = 0$ STEADY



$$\bar{a} = \underbrace{\frac{d\vec{V}}{dt}}_{\text{LAGRANG. ACCEL.}} \text{ along a trajectory / pathline} = \Big(\underbrace{V\frac{\partial V}{\partial s}}_{\text{convective}} + \underbrace{\frac{\partial V}{\partial t}}_{\text{eulerian local}}\Big)\hat{t} + \underbrace{\frac{V^2}{R}}_{\text{centripetal}}\hat{n}$$

If we measure directly LAGRANG. accel., we should make sure that the accelerometer is properly oriented! — we can measure a_t and a_n



along $\hat{t}$ Euler equation (INVISCID, INCOMPRESS. fluid)

for generic direction ℓ → ΣF... move ℓ

$$\frac{\partial}{\partial \ell}(p + \gamma z) = \rho a_\ell$$

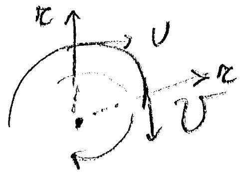
when $\ell \perp$ streamlines, $a_\ell = 0$

$p + \gamma z$ = constant along $n \perp$ stre...

11E

Press. distrib. in ROTATING FLOWS

apply Euler in direct. ⊥ velocity $\quad \frac{d}{dr}(p+\gamma z) = -\rho\frac{V^2}{r}$



$a = -\frac{V^2}{r}$

RIGID BODY ROTAT.

$V = \omega r$

$$p + \gamma z \boxed{-} \rho\frac{\omega^2 R^2}{2} = \text{CONST. along } r$$

⊥ streamlines.

note that these 2 equations are different ($\pm \frac{V^2}{2}\rho$)

so Bernoulli does not apply ⊥ to / ACROSS streamlines

Bernoulli!

apply Euler eq. along a streamline in steady flow

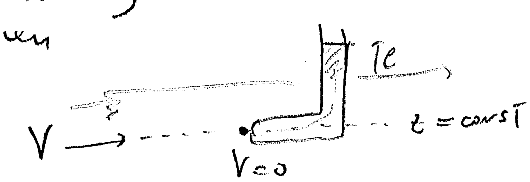
INVISCID, STEADY, INCOMPRESS.

$$p + \gamma z + \rho\frac{V^2}{2} = \text{CONST along streamline}$$

TANGENT.

$$\frac{p}{\gamma} + z \boxed{+} \frac{V^2}{2g} = \text{const}$$

important! apply Bernoulli along streamlines and compare points where boundary conditions are known



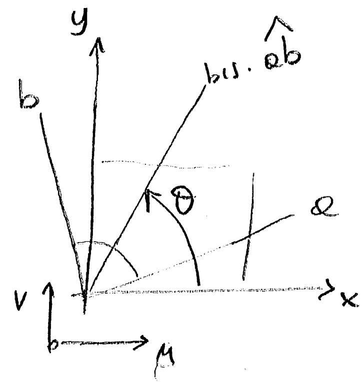
11F VORTICITY

$$\vec{\omega} = 2\vec{\Omega} = \left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right)\hat{i} + \left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right)\hat{j} + \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)\hat{k}$$

where $\vec{\Omega}$ is the ROTATIONAL velocity RATE

2D case

$$\Omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = \dot{\theta}$$



where $\dot{\theta} = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t}$

note

$$[\Omega_z] = \left[\frac{RAD}{s}\right]$$

IRROTATIONAL FLOWS → $\vec{\Omega} = 0$ in 2D → $\underbrace{\Omega_z = 0}_{\frac{\partial v}{\partial x} = \frac{\partial u}{\partial y}}$

Forced vortex

$V = \omega R$ ROTAT.

$\Omega_z = \omega$ RIGID BODY ROTAT.

Free vortex

$V = \frac{C}{r}$ IRROT.

$\Omega_z = 0$

if the flow is IRROT. we apply Euler eq. $\perp$ streamline

$$p + \gamma z + \rho\frac{V^2}{2} = \text{const}$$

but if we apply Euler // streamline → Bernoulli

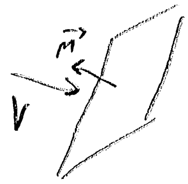
INCOMPRESS. STEADY, INVISCID, IRROT. → $\frac{p}{\gamma} + z + \frac{V^2}{2g} = \text{CONST}$ between any POINTS

OK // streaml. & $\perp$ streamline

11G] Rate of flow or volume flow rate
or discharge

$$Q = \int_A \vec{V} \cdot d\vec{A}$$

if $\vec{V} \perp \vec{A}$, $V \parallel \vec{m}$ $\Big\}$ $Q = \int_A V\, dA$



Note $[Q] = \frac{m^3}{s}$

if $V = \text{const}$ over A $\longrightarrow$ $Q = VA$

Given $Q \longrightarrow$ I can define

$$\overline{V} = \frac{Q}{A} \quad [m/s]$$

mean velocity

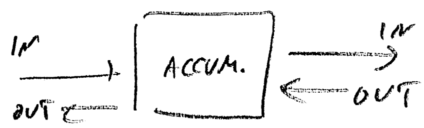
Similarly the mass flow rate

$$\dot{m} = \int_A \rho \vec{V} \cdot d\vec{A}$$

Definition of CONTROL VOLUME (fixed! in time)

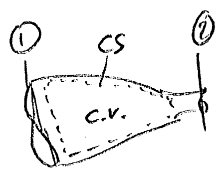
Continuity equation

$$\underbrace{\frac{d}{dt}\int_{c.v.} \rho\, dV_{ol}}_{\text{change in mass in the c.v.}} + \underbrace{\int_{c.s} \rho \vec{V} \cdot d\vec{A}}_{\text{net flux of mass through the c.s}} = 0$$



$$\frac{d}{dt} m_{c.v.} + \sum_{cs} \dot{m}_{OUT} - \sum_{c.s} \dot{m}_{IN} = 0$$

In pipes
if flow is steady:



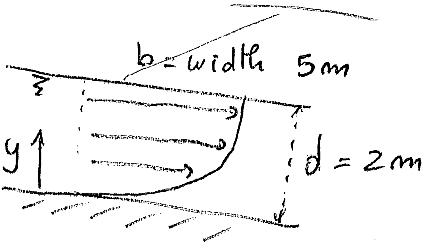
then $\dot{m}_1 = \dot{m}_2$

$Q_1 = Q_2$ $\Big\}$ in general $\sum_{c.s} Q_{in} = \sum_{c.s} Q_{OUT}$

$V_1 A_1 = V_2 A_2$

TO DO:
- Hydr. machines
- MANOMETRY

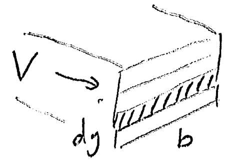
Example



water velocity in a channel

$$\frac{u(y)}{u_{MAX}} = \left(\frac{y}{d}\right)^{\frac{1}{2}}$$ DISTRIB. of velocity GIVEN ; $u_{MAX} = 3\,m/s$

2)) Discharge



$$Q = \int_A V\,dA \qquad \text{but} \quad dA = b\,dy = 5\,dy$$

$$Q = \int_0^d u(y)\,b\,dy = \int_0^d u_{MAX}\cdot\left(\frac{y}{d}\right)^{\frac{1}{2}}\cdot 5\,dy$$

$$= \frac{u_{MAX}\cdot 5}{d^{1/2}} \underbrace{\int_0^d y^{1/2}\,dy}$$

$$= \frac{5\,u_{MAX}}{d^{1/2}}\left[\frac{2}{3}y^{3/2}\right]_0^d = \frac{5\cdot\cancel{3}}{2^{1/2}}\left[\frac{2}{\cancel{3}}2^{3/2}\right] = 20\,\frac{m^3}{s}$$

What is the mean velocity $\bar{V} = \frac{Q}{A} = \frac{20\,m^3/s}{5\cdot 2\,m^2} = 2\,m/s$

What is the mass flow rate? $\dot{m} = \rho Q = 1000\,\frac{kg}{m^3}\,20\,\frac{m^3}{s} = \frac{20\,t}{s}$