

# **CEGE 3502 FLUID MECHANICS**

## **Lab Manual**

(Spring 2016 Edition)

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Course textbook:

Engineering Fluid Mechanics, 9<sup>th</sup> or 10<sup>th</sup> Ed.

Crowe, Elger, Williams, Roberson

John Wiley and Sons, New York, NY, 2009.

(all references to page numbers refer to the 9<sup>th</sup> edition of this textbook. Updates, hints, etc. will be posted online Fall, 2014. Please check: <http://www.ce.umn.edu/classpages/ce3502/>)





## TABLE OF CONTENTS

<b>Section</b>	<b>Page</b>
<b>General Instructions</b>	<b>5</b>
<b>Lab #1 Evaluation of Fluid Properties</b>	<b>8</b>
<b>Lab #2 Pressure &amp; Velocity Measurement</b>	<b>15</b>
<b>Lab #3 Applications of the Bernoulli Equation</b>	<b>19</b>
<b>Lab #4 Applications of the Momentum Theorem</b>	<b>25</b>
<b>Lab #5 Turbulence and Boundary Layer Demonstration</b>	<b>30</b>
<b>Lab #6 Measurements of Pipe Friction</b>	<b>32</b>
<b>Lab #7 Pipe Bends</b>	<b>36</b>
<b>Lab #8 Pipe Networks</b>	<b>38</b>
<b>Lab #9 Pump Performance &amp; Pump Sizing</b>	<b>43</b>
<b>Appendix A: Useful Equations</b>	<b>47</b>
<b>Appendix B: Integration Techniques</b>	<b>49</b>
<b>Appendix C: Minor Loss Coefficients</b>	<b>50</b>
<b>Appendix D: User's Manual for Pump Testing Stand</b>	<b>51</b>
<b>Appendix E: Moody Diagram</b>	<b>57</b>



# CE 3502 FLUID MECHANICS LABORATORY

## GENERAL INSTRUCTIONS

### INTRODUCTION

The Fluid Mechanics Laboratory is a substantial part of CE 3502 and is constructed to complement the lecture portion of the course. The labs are designed to provide the student with a physical understanding of the fundamental principles and basic equations of fluid mechanics. This understanding is gained through the application of “text book” concepts and equations to real problems.

### COURSE REQUIREMENTS

Attendance is required for all of the lab sessions. Each session, except one demonstration activity, requires the completion of a formal lab report. These reports are the basis of your final lab grade. With only eight assignments, each represents a substantial fraction of your total score.

### LABORATORY PROCEDURE

1. The student is to read the lab manual chapter assigned to each laboratory period BEFORE coming to the lab. Some labs contain thought questions or require that you perform some derivations before proceeding. In addition, the appendix contains useful information for many of the labs.
2. The class will work in groups to collect data for each set of experiments. It is desirable that groups are limited to three students, but some students will learn more efficiently in a smaller group, and some labs will require more students to work together to collect the data; it is still recommended that groups of no more than three work together to analyze the data. Some laboratory reports will be written by individuals and some submitted as group reports. This is detailed in #4 below.
3. In carrying out the experiments the following points are important:
  - a) Before starting any experiment, clearly define the goals. What question are you answering or what principle are you trying to demonstrate? What data is needed to solve the problem? The graduate instructor for each lab will help outline some necessary points, but it is ultimately it is each student’s responsibility to digest this information for the laboratory report, and, ultimately for class exams.
  - b) Identify the methods of measurement and instrumentation to be used. At the research stations, “play around” with the equipment so that you understand how the instruments work, what you are measuring, and how what you are measuring connects with the physics of the problems at hand.

- c) Perform the experiment. Collect data in a clear and organized fashion. **Be sure to note the units** for each measurement. Also, be sure to participate in the experiment rather than just recording data for your group.
  - d) Perform your calculations before leaving the lab if time allows. This can help you to better understand the concepts and equations presented in lab, and allow you to spot data collection errors while there is still time to take new measurements.
  - e) Write your group's formal report. This will be an easier task if done soon after lab while the concepts are fresh in your mind. Students who wait for the last minute often discover unanticipated problems.
4. Formal reports will be due one week after the experiment was performed. There will be some reports you will submit with your group (recommended: 3 people) and some reports you submit as an individual. These will be announced the day of the lab (if not sooner). In either case, feel free to work with classmates, but the final report should reflect the understanding and work performed by authors on the report. In no case should anything other than the raw data be copied from other students' work.
  5. Every report except one is graded on the basis of 100 possible points. The demonstration lab is much shorter, and carries the weight of 50 points. Absences can be made up only with permission of the instructor and, subsequently, by coordinating with the teaching assistant of your particular laboratory. Unfortunately, this cannot be done during another laboratory meeting period because the enrollment in each laboratory is so high.
  6. The policy for late laboratory reports is as follows: The first day the laboratory is late will carry a 5% penalty. Each subsequent week late (starting with the second day the laboratory is late will carry an additional 10% penalty.

## **FORMAL REPORT**

Formal reports should be completed with a computer, including all final calculations, charts, and data tables. The report should consist of the following sections:

1. Title page: Should contain:
  - (a) title of experiment
  - (b) laboratory section
  - (c) each author's name (one for individual reports, each group member up to 4 for group reports)
  - (d) Name of your TA
  - (e) Date on which the experiment was performed
  - (f) Signed and dated statement of honor (e.g. we did this work ourselves, etc.)

The names of other group members who aren't authors (i.e., those whose raw data is likely identical to the contents of this laboratory report) should be given at the bottom right-hand corner of the title page.

2. Purpose of the experiment: State the overall objective of the laboratory exercise first and then explain the objective of each particular experiment.
3. Apparatus and procedure: Briefly explain the procedure followed in the experiment. A concise explanation of the equations to be used in the computations should be given. Derivations of these equations are not necessary unless specifically indicated by the instructor or the manual. Much information regarding the instruments and procedures is included in the manual. You need not repeat it in the report. However, reference all information in your summary to specific page numbers in the manual or other reference material.
4. Tabulation of computed results: Computed results should be presented in a neat table with all rows and columns clearly defined. Specify the correct units at the heading of each column (row) in the table. **Specify all units clearly.**
5. Sample calculation: Show one complete set of typical calculations explaining step-by step how the results have been computed (for each type of calculation). One set of actual readings taken in the experiment should be used in the sample calculations. Units should be specified for all computed quantities. Handwritten calculations are OK here, but keep them neat and organized. **Specify all units clearly.**
6. Discussion: Discuss the results obtained and summarize your conclusions. This section should answer any questions that are stated or implied in the purpose. Assumptions made, difficulties encountered during the experiment, percentage error, and possible sources of error should also be included in the discussion. Some labs may require extensive discussion (a few paragraphs), while others may require just a few sentences. **Note: Questions to be answered may be in the text of this laboratory manual.** Look carefully to be sure you answer all the questions asked. **Specify all units clearly.**
7. Appendix: The raw data taken during the experiment, any handout sheets, and your response to the thought questions will form the appendix of the report. **Specify all units clearly.**

Your calculations and conclusions are the cornerstones of your report. Show units on all calculations (including intermediate steps) and in the results table(s), and always use an appropriate number of significant digits. Proper organization of the report and adequate presentation of the results will be given substantial weight in grading the work, as well.

The textbook referenced in this manual is:

Crowe, Elger, and Roberson, Engineering Fluid Mechanics, 9th ed, 2012 (John Wiley & Sons, New York).



**CE 3502 FLUID MECHANICS**  
Laboratory Experiment #1  
**EVALUATION OF FLUID PROPERTIES**

The solution of engineering problems in fluid mechanics requires numerical values for certain basic physical characteristics of the fluid of interest. These values are obtained by measurements using commercially available instruments and standard procedures. In this laboratory session, some fundamental properties of liquids will be measured. This lab is intended to familiarize the student with a basic understanding of several important properties used to describe fluids.

These are the fluid properties, and appropriate measurement devices, which we will use:

1. Density, specific gravity or specific weight using a hydrometer and a hydrometric or Westphal balance.
2. Absolute and kinematic viscosity using a Brookfield (rotational) viscometer and an Ostwald or capillary viscometer.
3. Surface tension using a tensiometer.
4. Density using a U-tube manometer.

General background information on the concepts involved may be obtained by reading Chapters 1 and 2 in the textbook.

**SPECIFIC GRAVITY** (Crowe et al., Ch. 2.1)

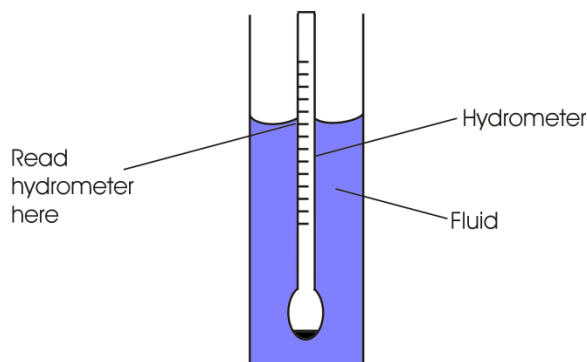


Figure 1: Hydrometer

$$\text{Specific Gravity (s)} = \frac{\rho_{fluid}}{\rho_{water}} = \frac{\gamma_{fluid}}{\gamma_{water}}$$

$$\text{Density } (\rho) = \frac{\text{Mass}}{\text{Volume}}$$

$$\text{Specific Weight } (\gamma) = \frac{\text{Weight}}{\text{Volume}} = \rho \cdot g$$

## Procedure:

### A. Hydrometers

Two selected fluids have been placed in glass containers together with hydrometers. The liquids used in these tests are among those listed in table A-4.

1. Take readings for both fluids. The scales read from the top down, and the readings must be divided by 1000 to obtain the specific gravity. The hydrometer should not be touching the sides of the container.
2. Calculate the specific weight ( $\gamma$ ) and the density ( $\rho$ ) in both SI units ( $\text{N/m}^3$  and  $\text{kg/m}^3$ ) and English units ( $\text{lb/ft}^3$  and  $\text{slugs/ft}^3$ ). Tentatively identify the two fluids in table A-4.

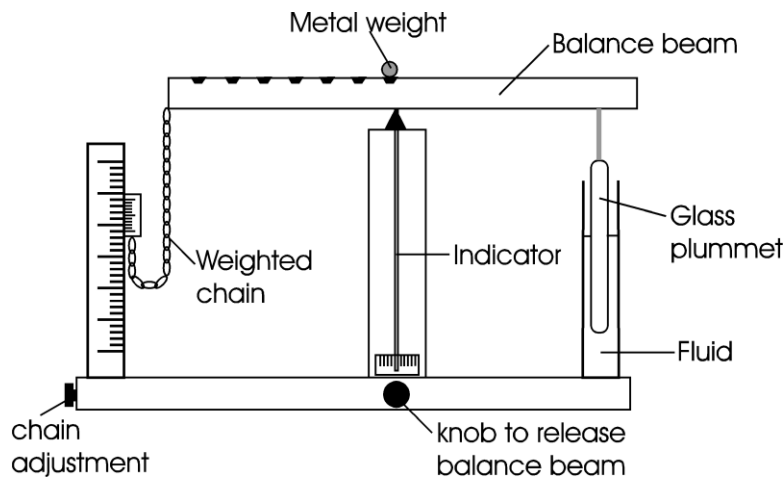


Figure 2: Westphal Balance

### B. Westphal Balance:

Use the Westphal Balance to obtain the specific gravity of Sigma-Aldrich Mineral Oil. Collect all the readings from the class and determine the mean and the standard deviation of the readings.

1. Check that the instrument is balanced.
2. Release the balance beam by turning the knob in the middle of the base.
3. Check that the glass plummet is not touching the sides of the container.
4. Adjust the metal weight on top and the weighted chain (using the knob on the left side) until the indicator is centered (on the scale at the bottom).
5. Read the specific gravity of the fluid. Read to 4 decimal places - the weighted chain uses a vernier scale.
6. Return the metal weight to the center and move the weighted chain back to the bottom of the scale.
7. Lock the balance beam.
8. Collect the values from all of the groups and calculate the mean and standard deviation. Calculate  $\rho$  and  $\gamma$  in SI and English units. Compare your values to the ones listed in the table and calculate %error.

## DYNAMIC (ABSOLUTE) VISCOSITY (Crowe et al., Ch. 2.4)

The dynamic (or absolute) viscosity of a liquid can be determined by using a rotational viscometer. The Brookfield viscometer rotates a sensing cylindrical element in a fluid and measures the torque necessary to overcome the viscous resistance to the induced movement.

The Brookfield LV model viscometers are provided with a set of four spindles and a narrow guard leg. The spindles are attached to the viscometer by screwing them onto the lower shaft. Spindles can be identified by the number on the side of the spindle nut. The viscometer is able to measure over a wide range of viscosity variation, since for a given spring deflection, the actual viscosity is proportional to both the size and shape of the spindle. The minimum viscosity range is obtained by using the largest spindle at the highest speed, the maximum range by using the smallest spindle at the lowest speed.

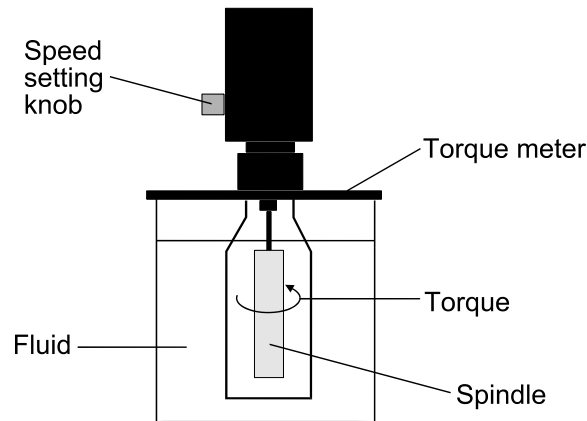


Figure 3: Brookfield Viscometer

### **Procedure:**

1. Check to see that the instrument is balanced.
2. Turn on the motor.
3. Set the speed to 12 rpm.
4. Wait 2-3 minutes until the reading stabilizes.
5. Hold the reading by pressing the clutch lever (located at the back of the instrument).
6. Stop the dial by turning the motor off, stopping the red mark in the window. Record the reading.
7. Find the constant K from the Brookfield factor finder chart (based on the model LV, spindle #1, and speed in rpm).  
$$\text{Viscosity, } \mu \text{ (in centipoise)} = \text{reading} * K$$
8. Repeat steps 3-7 for 30 and 60 rpm.
9. Find the average viscosity in both SI units ( $\text{N}\cdot\text{sec}/\text{m}^2$ ) and English units ( $\text{lbf}\cdot\text{sec}/\text{ft}^2$ ) ( $1 \text{ centipoise} = 2.09 \times 10^{-5} \text{ lbf}\cdot\text{sec}/\text{ft}^2 = 0.001 \text{ N}\cdot\text{sec}/\text{m}^2$ ). Compare your value with the value for Sigma-Aldrich Mineral Oil on page 15 of this manual (table A-4). Specify this. Calculate % error between your results and the value given.

## KINEMATIC VISCOSITY (Crowe et al., Ch. 2.4)

Kinematic viscosity of a liquid may be determined using an Ostwald-type capillary tube viscometer. The kinematic viscosity of the fluid is proportional to the time required for a given volume of liquid to flow through a capillary tube of given dimensions.

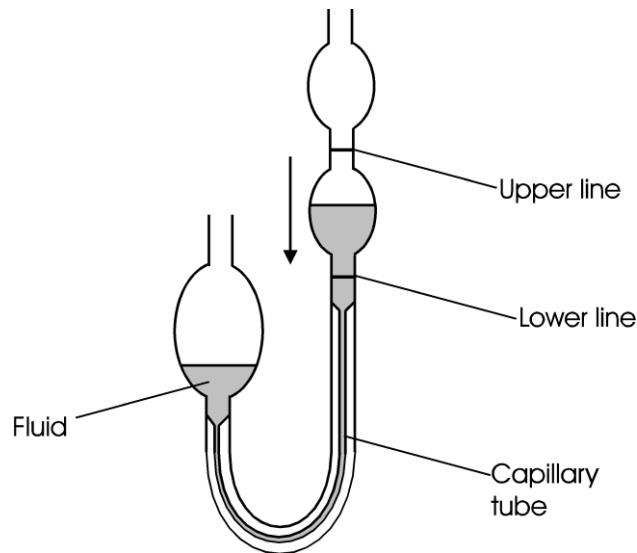


Figure 4: Capillary Viscometer

### Procedure:

1. Using the air line attached to the faucet, apply suction to the fluid until the fluid level is above the upper line (which is between the two small bulbs) of the capillary tube. Be careful not to suck any of the oil into the air line.
2. Measure the time it takes for the fluid level to drop from the upper line to the lower line.

Kinematic viscosity ( $\nu$ ) =  $K \cdot \text{time}$ .

$K$  (white) =  $1.215 \text{ mm}^2/\text{s}^2$  (Be careful with unit conversions!)

$K$  (yellow) =  $2.538 \text{ mm}^2/\text{s}^2$

Give the results in English units only ( $\text{ft}^2/\text{s}$ ).

3. For the Sigma-Aldrich Mineral Oil (white) check the measured value of the kinematic viscosity ( $\nu$ ) by comparing it with  $\nu_{\text{calc}} = \mu / \rho$  using  $\mu$  measured with the Brookfield viscometer and  $\rho$  calculated from the Westphal Balance.
4. Give the % error and comment.

Since the viscosity of liquids changes with temperature, it is important to read the temperature of the control bath at the time of viscosity evaluations

## **SURFACE TENSION** (Crowe et al., Ch. 2.6)

The surface tension of water at the interface with air will be determined by pulling a submerged wire ring through the free surface of a beaker of water using a special delicate force measuring system. The film of water pulled up from the surface by the ring will ultimately break as its tensile strength is overcome. A dial readout on the tensiometer gives the force acting on the film surface (in dynes per centimeter). The value read at the time of breaking or rupture is known as the apparent surface tension. For tap water this value will normally be about 60 dynes per centimeter ( $60 \times 10^{-5} \text{ N/cm}$ ), depending on the contaminants present in the water (pure water at room temperature has a value of  $73 \times 10^{-5} \text{ N/cm}$ ).

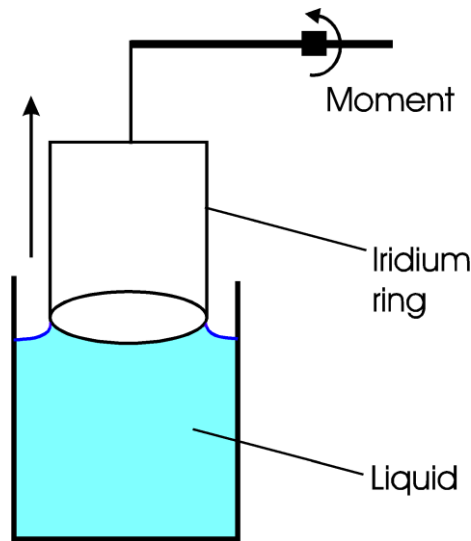
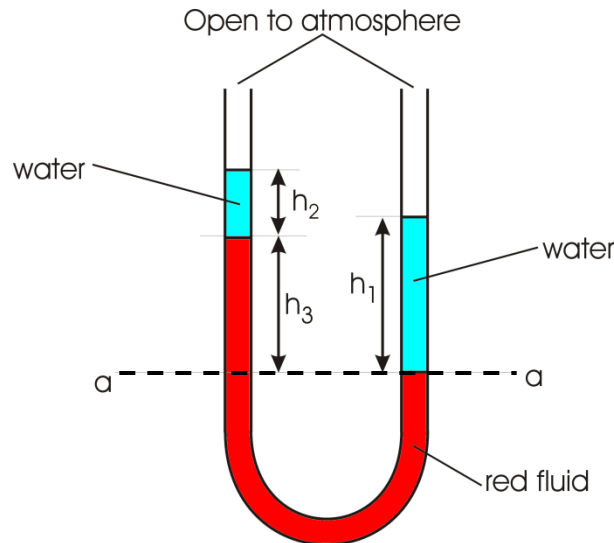


Figure 5: Surface Tensiometer

### **Procedure:**

1. Raise or lower the sample table (using the knob below it) until the ring is about 1/8 inch below the surface of the liquid.
2. Adjust the knob on the right side until the index and its image are exactly in line with the reference mark on the mirror. Be careful to keep the ring in the liquid by raising or lowering the sample table. Do not allow ring to contact the surface.
3. Turn the knob beneath the main dial on the front until the vernier scale reads zero on the outer scale of the dial.
4. Lower the sample table until the ring is in the surface of the liquid. At the same time, adjust the knob on the right to keep the index lined up with the reference mark on the mirror. Continue the 2 adjustments until the ring breaks the film at the surface of the water.
5. Repeat steps for tap water and tap water with detergent added (use two different tensiometers). Note: One instrument is manual and the other is automatic.
6. Convert your results to SI units (N/m).  $1 \text{ dyne} = 10^{-5} \text{ N}$ .

## U-TUBE MANOMETER (Crowe et al., Ch. 3.3)



Hydrostatic Principle :

$$P = \gamma \cdot h = \rho \cdot g \cdot h$$

Figure 6: U-Tube Manometer

### **Procedure:**

1. Using the U-tube manometer, calculate the density ( $\rho$ ) of an unknown liquid (red). Use SI units.
2. Calculate the pressures at the two interfaces between the water and red fluid ( $p_1$  and  $p_2$ ).

### THOUGHT QUESTIONS:

1. Find the kinematic viscosity ( $\nu$ ) for air and water ( $\nu = \mu/\rho$ ) in your textbook. Choose similar conditions (e.g., room temperature) and state them. Which is larger? Does the result surprise you? Why or why not?
2. Estimate, based on your own intuition, the approximate range of densities (or specific gravities or specific weights) of LIQUIDS that may be of interest to a civil, environmental, or geo- engineer. How much variation from the density of water would you expect? How do the results of your density measurements compare to this estimate? How likely are you to encounter a liquid with a  $SG = 10$ ? How about  $SG = 0.1$ ? Describe a situation in which the density difference between two common liquids is important in engineering applications.
3. Natural and man-made surfactants (similar to soap) are common impurities in lakes and oceans. Small scale wind waves (approximately the size you observe at the beach) on the water surface can be thought of as a balance between wind shear stress and surface tension, where the wave phase speed,  $c$ , for a given wavelength, is directly proportional to the square root of the surface tension,  $\sigma$ , as follows:

$$c \propto \sqrt{\sigma}$$

These small-scale waves are a dominant mechanism of transferring momentum and atmospheric oxygen into the water body, with faster waves possibly being more efficient

than slower waves for facilitating these generally desirable processes. Based on the results you obtained in lab, describe qualitatively how surfactants on the surface of the water body (i.e. “surface slicks”) will affect the transport of momentum and oxygen into the water body.

PROPERTIES OF SOME LIQUIDS AT 68°F IN ADDITION TO THOSE LISTED IN THE TEXTBOOK				
Liquid	Specific Gravity	Dyn. Viscosity <i>lbf-sec/ft<sup>2</sup></i>	Surface Tension <i>lbf/ft</i>	Vapor Pressure <i>psia</i>
Methyl Alcohol	0.8	$1.25 \times 10^{-5}$	0.00153	2.00
Kerosene	0.80-0.84	$3.8 \times 10^{-5}$	0.0016-0.0022	0.51
Castor oil	0.97	0.020		
Sea water	1.025	$2.25 \times 10^{-5}$		
Glycerin	1.260	$3.12 \times 10^{-2}$	0.0043	$2 \times 10^{-6}$
Olive oil	0.918	0.0018		
SAE 10 oil*	0.88-0.95	$6.0 \times 10^{-4}$	0.002	
SAE 30 oil*	0.88-0.95	$2.0 \times 10^{-3}$	0.002	
Sigma-Aldrich Mineral Oil	0.86	$5.56 \times 10^{-4}$		
* measured at 100°F				

Group #	Hydrometer S.G. ( )		Westphal S.G. ( )	Brookfield $\mu$ (Centipoise)	Capillary Viscometer Time (Sec)	Surface Tensiometer Force (dyne)	U-Tube $\rho$ (kg/m <sup>3</sup> )
	Red	Brown					
1							
2							
3							
4							
5							
6							

**CE 3502 FLUID MECHANICS**  
**Laboratory Experiment #2**

**PRESSURE & VELOCITY MEASUREMENT**

Measuring the pressure at a point in a flow field is one of the most common engineering fluid mechanics problems. Laboratories frequently use U-tube manometers, piezometric tubes, or pressure gages for measuring relatively steady pressures. When time-dependent (unsteady) pressures have to be recorded, it is recommended to use electronic pressure transducers, which are highly responsive to pressure oscillations.

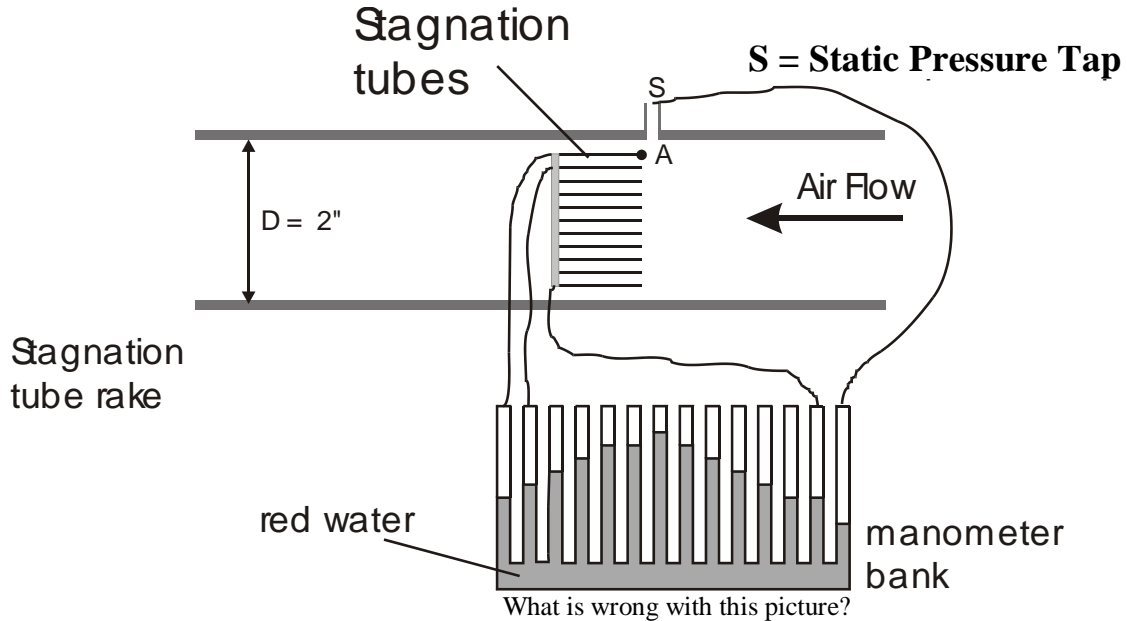
Closely related to pressure measurements are velocity measurements. Both are used to describe a flow field. The velocity at a point in a fluid stream may be measured by several methods. The most commonly used are the stagnation tube, Pitot tube, and the current meter. There are also some more delicate devices such as hot wires, Laser-Doppler systems (LDA) and particle image velocimetry (PIV). Additional information for some of these devices can be found in the textbook, Chapters 4 and 6. If you want additional information, please ask your instructors.

In order to demonstrate how the velocity profile in a flow can be measured and used to determine volumetric discharge, an experiment will be performed using stagnation tubes in an air stream flowing through a pipe. A two-inch diameter pipe has been fitted with a rake of stagnation tubes and a static pressure tap on the pipe wall. These are connected to a manometer bank filled with colored water, which will be used to determine a local velocity at the location of each stagnation tube within the cross-section of the pipe. The mean velocity is calculated from these local velocities by integrating a plot of local velocity vs. the square of the radial coordinate. This velocity is then compared with velocities determined using two additional approaches.

The two other measurements of flow velocity are obtained from (1) a Hastings meter and (2) a Venturi meter, which is a constricted section of the pipe with known geometry. A manometer connected to two static pressure taps on the Venturi meter (one at the constriction and one just upstream of it) is used to determine the pressure loss through the constriction, which can be related to the flow rate using a calibrated equation. Both measurements are described in more detail in the procedure section.



**STAGNATION TUBE RAKE** (e.g., Crowe et al., Chapter 4.5)



**Procedure:**

Steps 2 – 4 will be repeated for three different flow rates, which are controlled using a Rheostat at the end of the pipe. Use voltage settings of 60V, 80V, and 100V. **Note:** Do not operate the Rheostat at a setting greater than 110 Volts. This will cause problems with the manometers.

1. For each tube in the manometer bank, record the initial (no flow) static pressure (in cm). This value will not be part of any calculations, but it is important to see the effect of moving vs. static fluid.
2. Set the Rheostat to the appropriate voltage setting (60V, 80V, or 100V) and allow the system to reach steady state (i.e. when the manometer fluid levels stop fluctuating).
3. Record the appropriate stagnation and static pressures (cm) from the manometer bank.
4. **Important:** for each voltage setting, you will also take measurements of flow using the Hastings meter and the Venturi meter. **DO NOT** change the voltage setting until all data has been recorded. See the procedure section for these apparatus for instructions on taking those measurements.

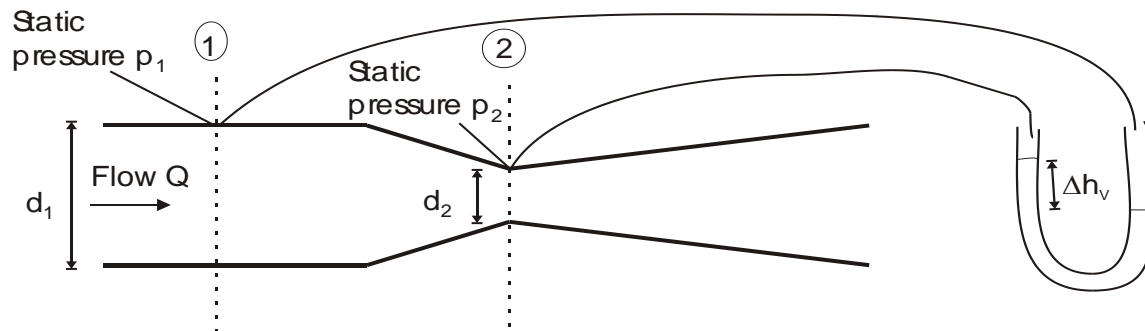
Note the shape of the profile observed in the manometer bank for the stagnation tubes. Is it similar to that shown in the diagram above? Why or why not? Discuss this in your report.

**Calculations:**

1. For each stagnation tube, calculate the pressure difference  $\Delta h$  (cm water) between the static and stagnation pressures at that location, i.e.  $\Delta h = h_{\text{static}} - h_{\text{stag}}$ .

- Calculate the local velocities at each stagnation tube from  $u = \sqrt{2g\Delta h \frac{\gamma_w}{\gamma_{air}}}$ . This can be derived from the Bernoulli equation, which will be discussed in the near future. Check your units to make sure the result is dimensionally consistent and reasonable!!!
- Plot the velocity distribution,  $u(r)$  vs.  $r$ , where  $r$  is the radial location of the tube (distance from the center of the pipe).
- Discuss the shape of the velocity profile as a consequence of viscous wall shear stress.
- Plot  $u(r)$  vs.  $r^2$ .
- Find the volumetric flow rate  $Q$  by numerical or graphical integration of the area under the  $u(r)$  vs.  $r^2$  curve (multiply this area by  $\pi$ ). Show in your report why this integration yields the volumetric flow rate within the pipe. Hint: refer to the calculus concept of finding volume under the surface generated by rotation of a curve about an axis.
- Calculate the mean velocity of flow in the cross section:  $\bar{V} = Q/A$
- Determine which of the stagnation tubes in the rake most nearly measures the mean velocity
- Repeat these calculations (Steps 1 – 8) for the second and third flow rates.

### VENTURI METER (Crowe et al., Ch. 4.6)



#### **Procedure:**

- Using the manometer connected to the Venturi meter, record the static pressure at points (1) and (2) in mm. Do this for each flow setting (60V, 80V, 100V) at the same time as the measurements for the stagnation rake and Hastings meter.

#### **Calculations:**

- Calculate the height difference between the two points,  $\Delta h_v = h_2 - h_1$ .
- Determine the flow rate from the following calibrated equation (which is also found on a chart on the side of the manometer bank):  
 $Q = 2.0786(\Delta h_v)^{0.5}$ , where  $\Delta h_v$  is in mm and  $Q$  is in L/s.
- Convert your value for  $Q$  to  $m^3/sec$ .

## HASTINGS METER

### **Procedure:**

1. Record the flow rate  $Q$  (in  $\text{m}^3/\text{min}$ ) directly from the Hastings meter. Note that a reading of 123 =  $1.23 \text{ m}^3/\text{min}$ . Do this for each flow setting (60V, 80V, 100V) at the same time as the measurements for the stagnation rake and the Venturi meter.

### **Calculations:**

1. Convert your reading to  $\text{m}^3/\text{sec}$ .
2. Calculate:  $K = \frac{Q_{\text{Venturi}}}{Q_{\text{Hastings}}}$

This is the correction factor for the Hastings meter. Save this value as it will be used in future labs.

3. For each voltage setting, compare the flow rates determined from the three methods in this experiment (stagnation rake, Venturi meter, and Hastings meter). Which method do you think is the most accurate? Why? What are some potential sources of error in these measurements?

### THOUGHT QUESTIONS:

1. Before performing the experiment, estimate a reasonable average air velocity for the pipes used in this lab. Is it 1 m/s? 10 m/s? 100 m/s? 1000 m/s? More? Somewhere in between? Convert to more familiar units (e.g. miles per hour) if you have trouble estimating speeds in SI units. In a 2-inch diameter pipe, what flow rate corresponds to your estimated velocity? The average student apartment in the Twin Cities has less than  $1000 \text{ ft}^2$  of finished space (usually MUCH less). Assuming 8-foot ceilings (or 10-foot if you like round numbers), estimate the time required to fill the average student apartment with a fluid at the flow rate you estimated for the air pipes previously. How long would it take to fill a five-gallon bucket? Does your result seem reasonable? Is your estimated flow rate substantially different (on an order of magnitude basis) from the results obtained during the experiment? If so, is the discrepancy a result of a misestimate or a miscalculation?

**CE 3502 FLUID MECHANICS**  
**Laboratory Experiment #3**

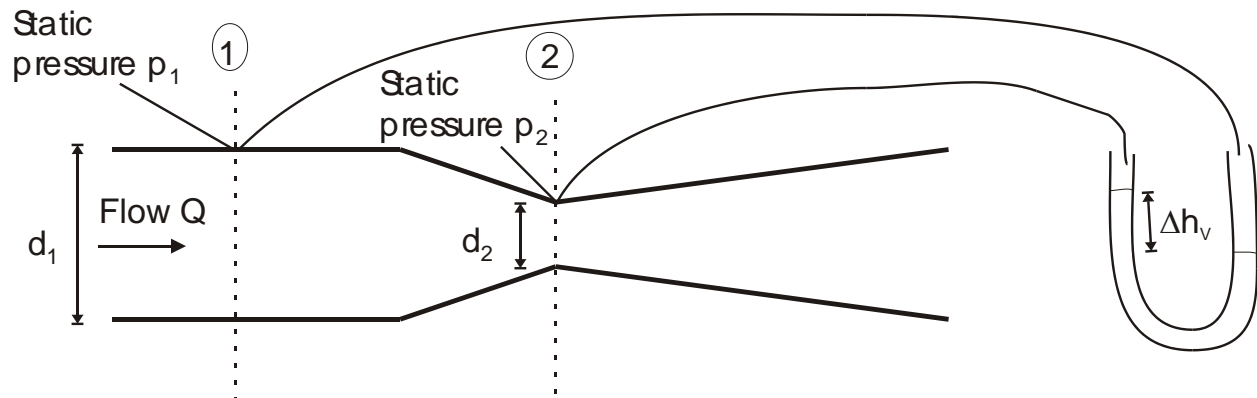
**APPLICATIONS OF THE BERNOULLI EQUATION**

The Bernoulli Equation for steady, incompressible, inviscid flow along a streamline is given in Sec. 4.5 of your textbook. Some illustrations of its use, particularly in flow measurement, are also presented in Chapter 4. The Bernoulli equation for irrotational flow is derived in Chapter 4.7 of the textbook. Its applications are also discussed.

***EXPERIMENT***

The experiment deals with the measurement of discharge in several flows by measuring a pressure difference and applying the Bernoulli equation and the equation of continuity to calculate the discharge. A comparison will be made between the values calculated from Bernoulli ('ideal' flow) and those obtained experimentally through the use of a previously calibrated device ('real' flow). You will be assigned three types of flow meters for your experiment: a Venturi meter, an orifice meter, and a sluice gate.

**1. VENTURI METER**



$d_1 = 2$  inches

$d_2 = 1$  inch

- Equations:** (1) Continuity Equation:  $Q = V_1 A_1 = V_2 A_2$   
(2) Bernoulli's Equation between points 1 and 2:

$$\frac{p_1}{\gamma_{\text{air}}} + z_1 + \frac{V_1^2}{2g} = \frac{p_2}{\gamma_{\text{air}}} + z_2 + \frac{V_2^2}{2g}$$

These equations can be solved simultaneously to determine velocity (and discharge) from a change in pressure across a manometer. **Derive an expression for  $V_2$**  as a function of a pressure change that doesn't contain  $V_1$ . You will conclude that both unknown velocities can be determined given a pressure change and values for  $A_1$  and  $A_2$ . Show this in your report. Note also that this is the 'ideal' flow, as it does not take into account any losses (e.g. friction).

## Procedure:

**Use Pipe 1 or Pipe 2.** For three different speeds (60, 80, 100 Volt settings on the rheostat):

1. Measure  $\Delta h_v$  from the large manometer bank for the Venturi Meter. This will be used to determine the ideal flow rate,  $Q_{ideal}$ .
2. Measure  $\Delta h_{mean}$  from the stagnation tube rake; this is the difference in pressure between the static tap and a stagnation tube assumed to correspond to the mean velocity. This will be used to find the mean (real) velocity and flow rate,  $Q_{real}$ .

$$\Delta h_{mean} = h_{static} - h_r, \text{ where } h_r \text{ is read from the stagnation tube at } r = 0.85'' \text{ for pipe \#1} \\ \text{and } r = 0.75'' \text{ for pipe \#2}$$

If your data from Lab #2 showed that a different radius matched the mean velocity, then use that radius instead of the one above.

3. Read the flow rate from the Hastings electronic flow meter. This provides a second measurement of  $Q_{real}$ .

$$Q_{real} = Q_{Hastings} * K_C \frac{\text{m}^3}{\text{min}}$$

where K is the correction factor computed in Lab #2 or given by the instructor.

## Calculations:

Part 1:

1. Calculate the pressure loss through the Venturi meter,  $\Delta p = \gamma_w \Delta h_v$
2. Calculate  $Q_{ideal}$  by using  $\Delta p$  in the equation for  $V_{ideal}$  that you derived from the Bernoulli and continuity equations ( $Q_{ideal} = V_{2,ideal} * A_2 = V_{1,ideal} * A_1$ ). Do NOT use the equation from the side of the manometer bank, it is a calibrated equation.
3. Calculate  $Q_{real}$  from  $Q_{real} = V_{real} * A$ .  $V_{real}$  is the mean velocity calculated by using  $\Delta h_{mean}$  in the equation for the local velocity  $u$  at a point in the stagnation rake (see previous lab).
4. Calculate the discharge correction coefficient.  $C_v = \frac{Q_{real}}{Q_{ideal}}$
5. Calculate the Reynolds number for pipe flow measured at the throat of the Venturi meter.

$$Re = \frac{V_2 d_2}{\nu_{air}} \text{ where } \nu_{air} \text{ is the kinematic viscosity of air, and } \nu_2 = \frac{Q_{real}}{A_2}$$

6. Plot your results on the graph on page 24.

Part 2:

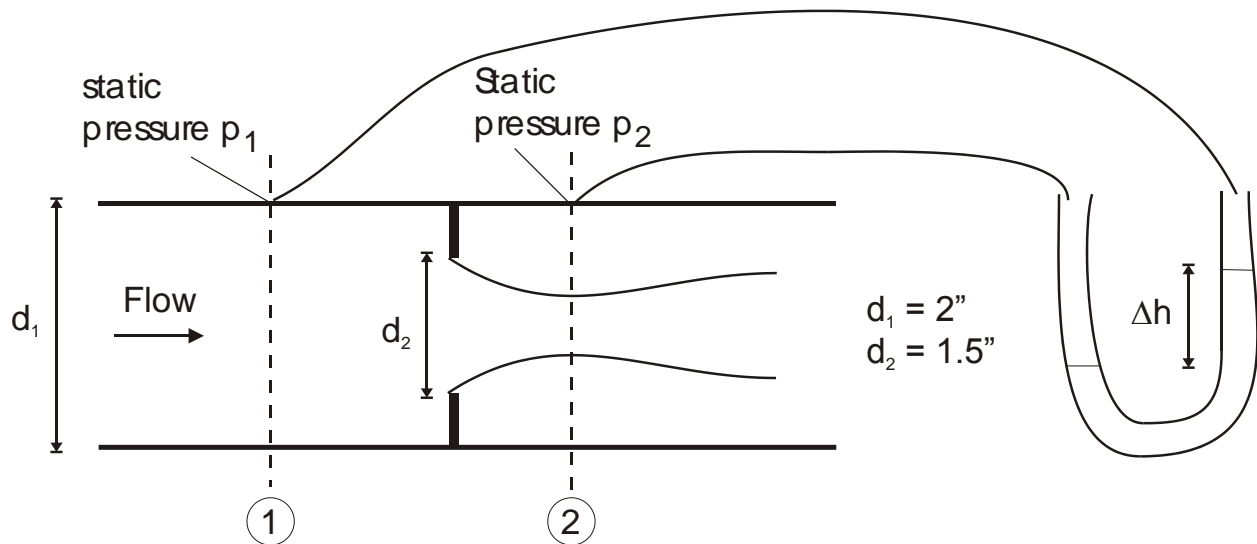
1. Using  $Q_{real}$  from the Hastings meter, calculate the discharge correction coefficient:

$$C_v = \frac{Q_{real}}{Q_{ideal}}$$

2. Calculate the corresponding Reynolds number.
3. Plot your results on the same copy of the graph on page 24.

Why is  $Q_{real}$  different than  $Q_{ideal}$ ? Which is greater? Why? Address this in the discussion section of your report.

## 2. ORIFICE METER (e.g., examples in Chapter 13.2 and 13.4)



The contraction coefficient for this set-up is:  $C_C = 0.6 + 0.4 \left( \frac{A_2}{A_1} \right)^2$

### Procedure:

Use **Pipe 3**. For three different speeds (60, 80, 100 Volt settings on the rheostat):

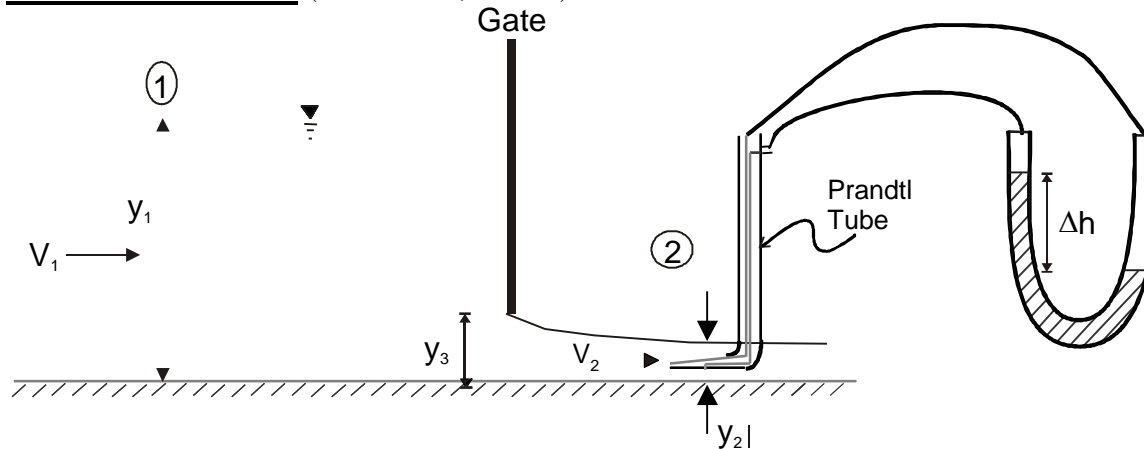
1. Measure  $\Delta h_o$  from the manometer connected to the orifice meter. This will be used to find  $Q_{ideal}$  using the equations given below.
2. Get  $Q_{real}$  from the Hastings Mass Flow Meter:

$$\text{e.g. } Q_{real} = Q_{Hastings} * K \frac{\text{m}^3}{\text{min}} \quad \text{where } K \text{ is the correction factor (1.1180)}$$

### Calculations:

1. Calculate  $Q_{ideal}$  using the orifice equation  $Q = \frac{C_C A_2}{\sqrt{1 - C_C^2 \left( \frac{A_2}{A_1} \right)^2}} \sqrt{2g(Dh_o) \frac{\rho_{water}}{\rho_{air}}}$
2. Calculate the discharge correction coefficient  $C_v = \frac{Q_{real}}{Q_{ideal}}$
3. Calculate the Reynolds number
4. Plot  $C$  vs.  $Re$  on Figure 11.26 on page 23, using  $C = \frac{C_v C_C}{\sqrt{1 - C_C^2 \left( \frac{A_2}{A_1} \right)^2}}$

### 3. SLUICE GATE (Crowe et al., Ch. 6.4)



\*The Prandtl tube measures both stagnation pressure and static pressure. It has separate pressure taps and separate outlets for each. Look at the Prandtl tube to locate the pressure taps.

#### Procedure:

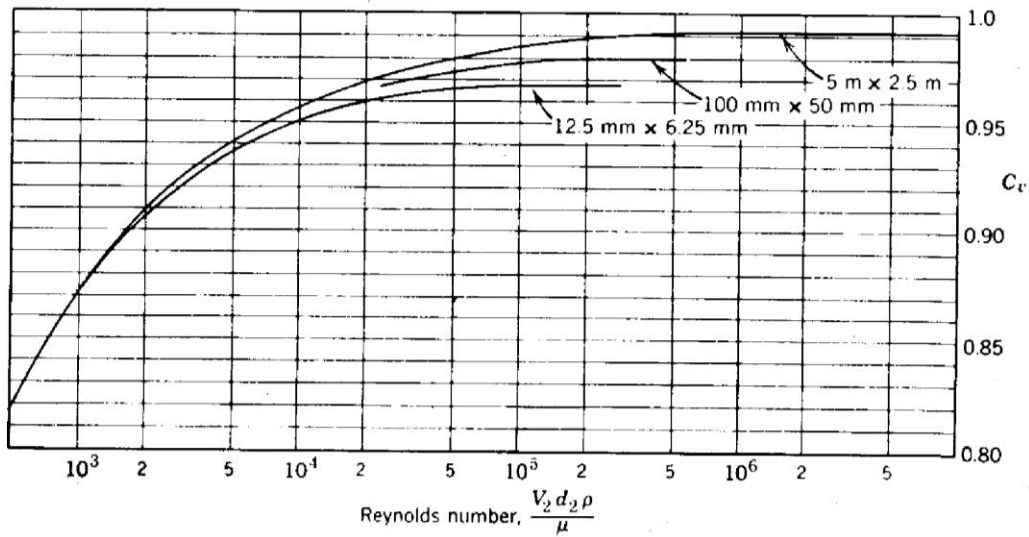
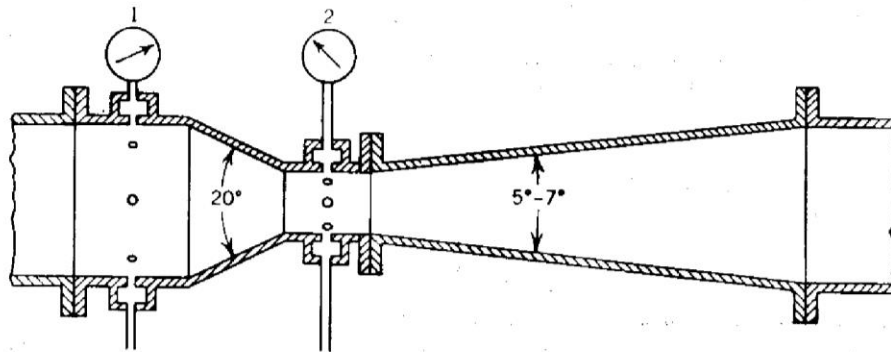
1. Measure  $y_1$  and  $y_2$ . These will be used to determine the ideal flow rate,  $q_{ideal}$ .
2. Measure  $\Delta h$  from the Prandtl tube manometer, this will be used to find the mean velocity

#### Calculations:

1. Calculate  $q_{ideal}$  (flow per unit width) using the Bernoulli and continuity equations applied to the sluice gate.
2. Using  $\Delta h$  from the Prandtl tube manometer, calculate the mean (real) velocity downstream of the sluice gate:  $\bar{V} = \sqrt{2gDh}$ . Then  $q_{real} = \bar{V} * y_2$
3. Calculate  $C_v = \frac{q_{real}}{q_{ideal}}$

#### THOUGHT QUESTIONS:

1. Explain the difference between static and stagnation pressure. Which is larger? And by how much? Why? Hint: consider a force balance on a small element of fluid along the pipe wall and on a small element of fluid just upstream of a stagnation tube.
2. As you've seen, static pressures increase with decreasing velocity. Qualitatively describe how this principle explains how an airplane wing generates lift. How might such an analysis be useful in designing certain types of hydraulic structures where the long-term effects of static pressure differences may cause damage and/or failure of the structure? Such problems can be minimized by proper consideration of the flow properties or by overly strengthening the structures. Which approach is most desirable from the standpoint of safety? From the standpoint of cost? From the standpoint of long-term durability and maintenance concerns? Why?
3. Given your experience with flow acceleration through the throat of a Venturi meter and under a sluice gate, consider an atmospheric flow over a hill where the flow far above the hill is not affected by the hill. How might such an analysis be useful in designing hilltop structures that are sensitive to wind forces? How might a continuity analysis be useful in considering wind acceleration between the large buildings in downtown Minneapolis? Describe safety concerns with such a deflection of atmospheric flows in an urban area.



**Fig. 11.23** Venturi meter and coefficients.

From: Elementary Fluid Mechanics, 6th Ed., Vennard and Street, 1982, John Wiley & Sons, New York, NY, page 527.



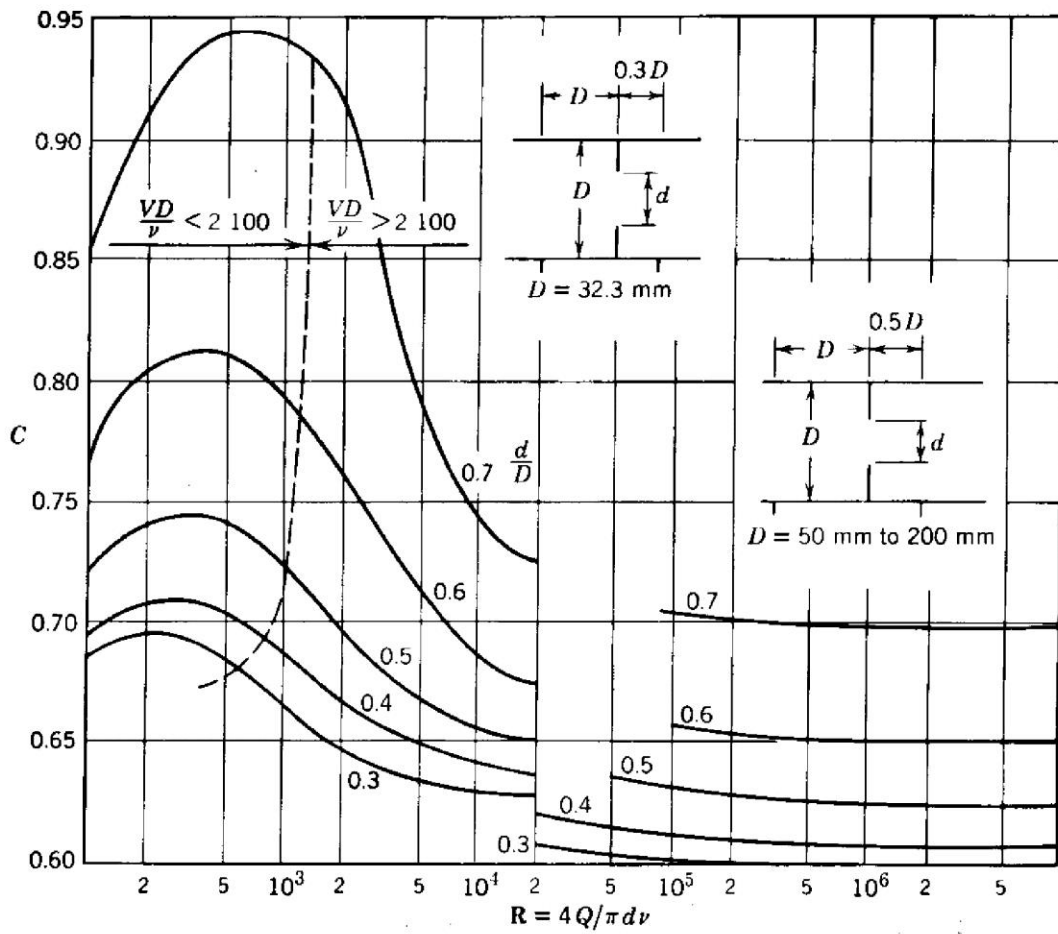


Fig. 11.26 Orifice meter coefficients.<sup>29</sup>

From: Elementary Fluid Mechanics, 6th Ed., Vennard and Street, 1982, John Wiley & Sons, New York, NY.

**CE 3502 FLUID MECHANICS**  
**Laboratory Experiment #4**

**APPLICATIONS OF THE MOMENTUM THEOREM**

The momentum theorem states that the rate of change in momentum of a fluid “particle” or control volume is equal to the sum of the forces on the fluid. This principle and its resulting equations may be used to determine the fluid accelerations where the external forces are known. Conversely, if the accelerations are known beforehand, the applied forces may be determined for design purposes.

**FLAT PLATE FREE JET:** (use English units)

A very simple example of the application of the principle is a free jet directed perpendicularly to a plane surface. The force on the flat plate is  $F = \rho QV$ , where  $\rho$  is the density of the fluid,  $Q$  is the volumetric flow rate and  $V$  is the jet flow velocity. **A detailed proof of this equation, starting with the Momentum Eqn. given in your textbook (Eq. 6.5; see also Appendix A of the lab manual), should be included in the report.** *Hint: consider only forces and the resulting momentum in the vertical direction using a control volume approach. Neglect gravity.*

In the laboratory experiment, two different approaches are used to estimate the force of the free jet,  $F_{jet}$ : (1) from the momentum equation (‘calculated’  $F_{jet}$ ) and (2) from direct measurement (‘measured’  $F_{jet}$ ). In the first approach, the flow rate is calculated by discharging a given weight of fluid (26 lb) over a measured time interval. This weight flow rate (lb/sec) is converted to a volumetric flow rate (ft<sup>3</sup>/sec). Velocity is obtained directly from the volumetric flow rate, given the cross-sectional area of the jet. The force that the jet exerts on the disk is then calculated from the momentum equation. In the second approach,  $F_{jet}$  is measured directly on a beam balance supporting the disk. You will then compare the ‘calculated’ and ‘measured’ values of  $F_{jet}$ .

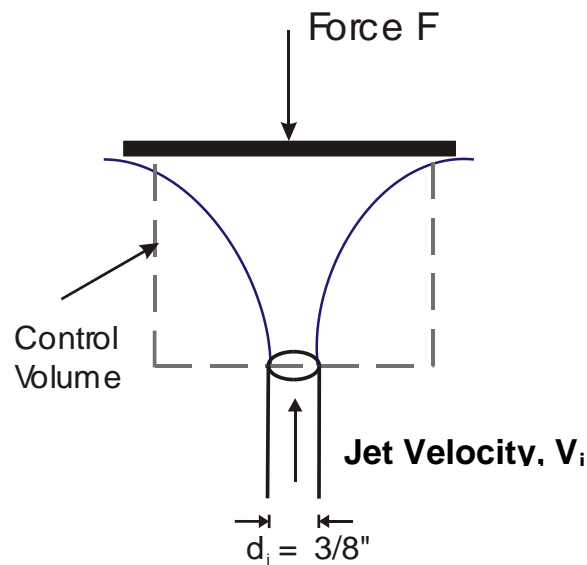


Figure 1: Jet discharging vertically against a flat plate

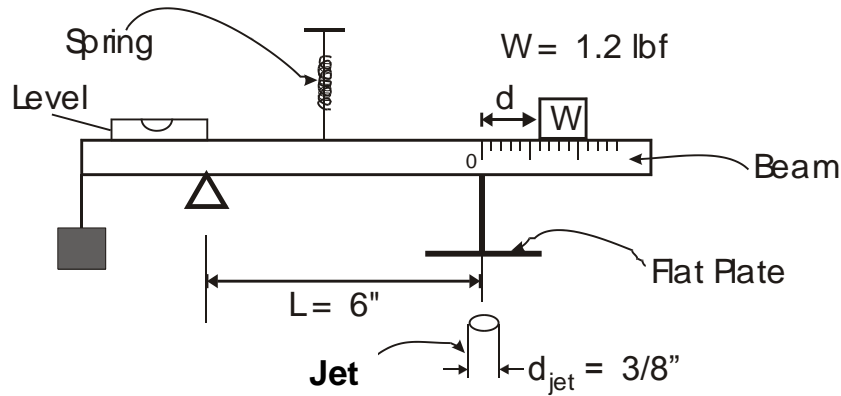


Figure 2: Flat Plate Free Jet Dynamometer

### Procedure:

Note: You will take 4 sets of measurements at two different values of  $d$  (1.5 inches and 3.0 inches), for a total of 8 measurements.

1. Turn on the pump (silver switch)
2. Open the water valve (big white knob)
3. Set the weight  $W$  at the appropriate distance  $d$  (1.5 inches for the first trial)
4. Adjust the water jet flow rate until the beam is balanced
5. Measure the accumulation time for 26 lbs of water (from the time you close the control knob on the side of the bucket until the balance tips). Repeat this measurement 4 times. This is done to reduce experimental error.
6. Repeat steps 3-5 for a setting of  $d = 3$  inches. You should have 8 total measurements.
7. Turn off the flow.

### Calculations:

1. The 'measured' value of  $F_{\text{jet}}$  is found for each value of  $d$  using moments:  $F_{\text{jet}} \cdot L = W \cdot d$
2. The 'calculated' value of  $F_{\text{jet}}$  is found from the momentum equation, using a jet diameter  $d_{\text{jet}}$  of  $3/8$ " (you should have 4 values  $F_{\text{jet}}$  for each of two values of the offset distance  $d$ )
3. Calculate the percent error between the 4 'calculated' and 'measured' values of  $F_{\text{jet}}$  for each value of  $d$ . Discuss these results in your report.

### HEMISPHERICAL PLATE FREE JET: (use SI units)

Another application of the free jet uses a hemispherical surface. As above, **derive the forces on the hemispherical plate,  $F = 2\rho QV$ , starting from Eq. 6.5 or the momentum equation given in Appendix A.** Why is the factor of 2 present in this example? *Hint: Assume that the water is diverted directly downward from the hemispherical plate with a flow and velocity equal to the upward flow and velocity. Neglect gravity. Remember that the jet is deflected downward (i.e. negative  $z$  direction) rather than outward as in the previous experiment*

In this laboratory experiment,  $Q$  will be computed based on time to collect a given water volume. Velocity  $V$  may be obtained as  $Q/A$  if nozzle area is known. The force of the jet may be calculated and compared to the weight force used to balance the jet.

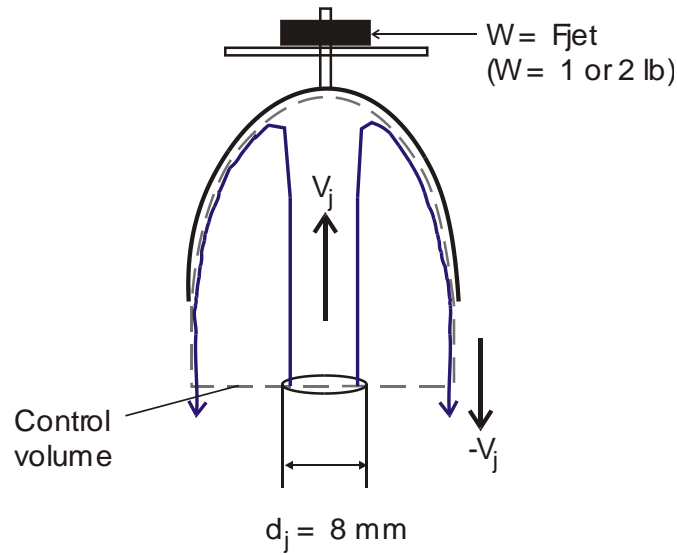


Figure 3: Diagram of hemispherical plate free jet

**Procedure:**

1. Put the 1lb weight on the disk.
2. Turn on the pump.
3. Open the metal valve and adjust the flow rate until the line on the disk is lined up with the pointer.
4. Close the drain (check by hand that the drain plug is firmly in place) and wait until the water level on the side rises to 0, then record the time required for the basin to fill to 40L.
5. Open the drain and let the basin empty.
6. Add another 1 lb weight (for 2lb total), readjust the flow rate, and repeat experiment.
7. Close the valve and turn off the pump.

**Calculations:**

1. Calculate  $Q$  (L/sec) from the time it takes to fill the basin, and then use your equation derived from the momentum equation to calculate  $F_{\text{jet}}$  (jet diameter = 8 mm). This is the ‘calculated’ value of  $F_{\text{jet}}$ .
2. The ‘measured’ value of  $F_{\text{jet}}$  is simply the weight placed on top of the disk (1 lb or 2 lb).
3. Compare the two values and calculate the percent error between the calculated and measured values of  $F_{\text{jet}}$ .
4. Discuss these results in your report.

**SLUICE GATE:** (use English units)

A third experiment involves determination of the force on a sluice gate in a 6-inch wide experimental channel. The water surface elevations upstream and downstream from the sluice gate are to be measured. The force on the gate can then be computed and compared with that obtained from the measured pressure distribution. The downstream velocity is also measured, at a single point, using a Prandtl tube.

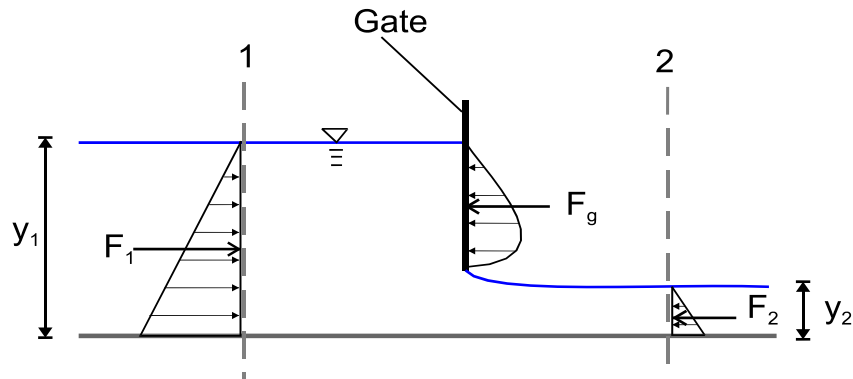


Figure 4: Sluice Gate Force Diagram

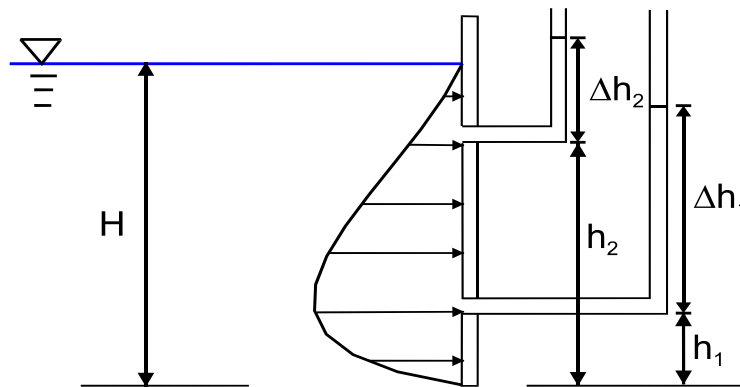


Figure 5: Close-up of Sluice Gate

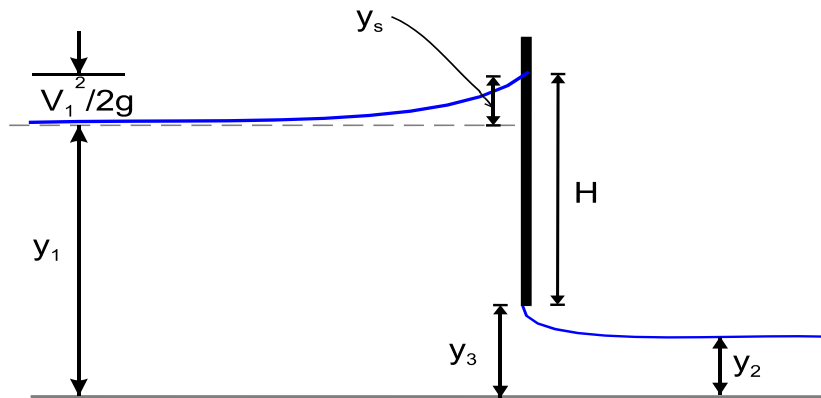


Figure 6: Diagram of Sluice Gate Depths

### Procedure:

1. Measure  $y_1$ ,  $y_2$ ,  $y_3$ , where  $y_1$  is the upstream flow depth,  $y_2$  is the downstream flow depth, and  $y_3$  is the distance between channel bottom and gate bottom.
2. Measure  $\Delta h$  from the manometer connected to the Prandtl tube. This will be used to find the flow velocity,  $V_2$ .
3. Measure the static pressures along the gate's surface using the manometers connected to taps in the gate. You will record the height of the water in each tube. You will also need to record the height of each tap above the bottom of the gate; this is obtained from a sheet on the side of the sluice gate.

### Calculations:

1. Velocity and flow calculations:
  - a. Calculate the downstream velocity using the Prandtl tube.  $V_2 = \sqrt{2g\Delta h}$
  - b. Use  $V_2$  to calculate the flow rate.  $Q = V_2 A_2$   
Discuss possible errors in using this method of calculating  $Q$ .
  - c. **Derive an equation for  $V_2$  using the Bernoulli and continuity equations between sections 1 and 2 in Figure 4.** Calculate and compare this velocity with the measured value and give the percent error.
  - d. Discuss these results.
2. Force calculations:
  - a. From the momentum equation ( $\sum \text{Forces} = \Delta \text{Momentum}$ ) **derive an expression for the force on the gate** and calculate  $F_1$  and  $F_2$ . Solve for  $F_{\text{gate}}$ .
  - b. Calculate the static pressures along the gate's surface. The pressure,  $P$ , at each tap location is related to the height of water in the manometer using hydrostatic principle ( $P = \gamma_w \Delta h$ ). Figure 5 illustrates the calculation of  $\Delta h$  for each tap.
  - c. Calculate  $F_{\text{gate}}$  using the pressure profile. Graph the pressure profile with  $p$  vs. vertical distance  $h$  along the gate.  
Assume pressure is zero at  $h = 0$  and at  $h = H$ .  
 $H = \text{maximum wetted length} = y_1 + y_s - y_3$ , where  $y_s = (V_1)^2 / 2g$ .  
-- Assuming  $V_1 = 0$ , then  $y_s = 0$   
 $F_{\text{gate}}$  is equal to the total area under the graph multiplied by the channel width.  
(channel width = 6 inches)
  - d. Compare the two values of  $F_{\text{gate}}$  and give the percent error. Discuss the results.

### THOUGHT QUESTIONS:

1. Consider the question about hydraulic structures in the previous section in terms of the dynamic pressures/forces studied in this experiment. How can the shape and orientation (with respect to the flow direction) of a hydraulic structure affect the forces that it is required to bear?
2. Describe how such a momentum analysis should be implemented in the design of tall buildings that are sensitive to wind stresses.

# CE 3502 FLUID MECHANICS

## Laboratory Experiment #5

### TURBULENCE AND BOUNDARY LAYER DEMONSTRATIONS

In this laboratory experiment, you will explore some different techniques for visualizing fluid flow and you will be able to observe laminar and turbulent flows and attached and separated flows around objects. You will also make observations and measurements in a cavitating system. As you progress through the different parts of this laboratory, please fill out this form and hand it in to your TA at the end of the class period. This is the only write-up you will have for this laboratory.

#### A. Turbulent and laminar flows

##### 1. Reynolds Experiment

Demonstrate laminar and turbulent flow regimes with a single streamline of dye in a pipe positioned downstream of a reservoir in the channel. Fill the reservoir with water. Open the valve at the end of the pipe to start the water flowing, and release a very thin stream of dye into the pipe. Adjust the water flow rates to exhibit laminar, turbulent, and transition flow regimes.

Time a dye streak to compute the flow velocity. Calculate a Reynolds number for laminar, transitional, and turbulent flow. Do these values match the pipe flow Reynolds number for the transition between flow regimes (see textbook p. 317)?

Pipe Inside Diameter = 1.0 cm.

Kinematic Viscosity of Water ( $\nu$ ) =  $1.06 \cdot 10^{-5}$  ft<sup>2</sup>/sec (at 70°F)

$$Re = \frac{V_{ave} \cdot D}{\nu}$$

Time (laminar): \_\_\_\_\_

$V_{ave}$ : \_\_\_\_\_

Re:

Time (transition): \_\_\_\_\_

$V_{ave}$ : \_\_\_\_\_

Re:

Time (turbulent): \_\_\_\_\_

$V_{ave}$ : \_\_\_\_\_

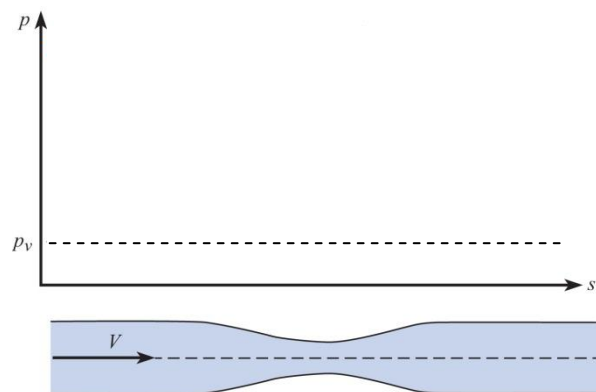
Re:

## 2. Cavitation Experiment

Your TA will perform a demonstration by repeating so that you can answer the questions below and on the next page.

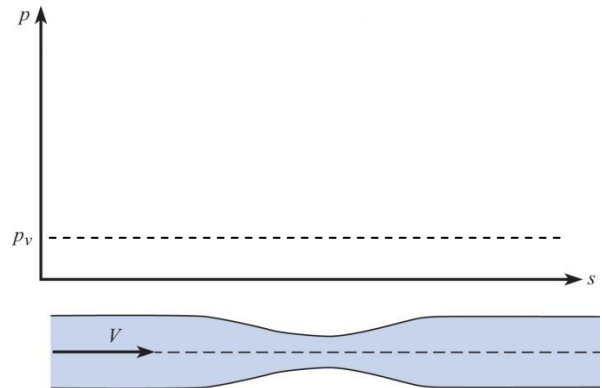
1. Open the ball valve (right end) fully then close the inlet diaphragm valve (left end) fully.
2. Close the flow control valve on F1-10. Switch on the F1-10 then slowly open the flow control valve on F1-10 until it is fully open.
3. Slowly open the inlet diaphragm valve at the left end of the F1-28 and allow water to flow until the clear acrylic test section and flexible connecting tubes are full of water with no air entrained.
4. Continue to open the inlet valve slowly until fully open to obtain maximum flow through the system.

**Low flow.** Record your observations (visual and audible) for conditions where there is low flow and no cavitation. Record the pressure for each gage. Sketch your estimate of the pressure vs. position in the plot below

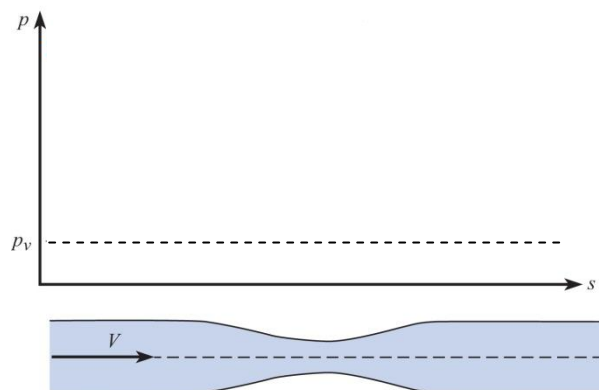




**Insipient Cavitation.** Record your observations (visual and audible) for conditions where there is *insipient* cavitation. Record the pressure for each gage. Sketch your estimate of the pressure vs. position in the plot below and to the left and label it so that it is distinct from the first plot. Sketch the appearance of the flow in the diagram below.



**Maximum Flow** Record your observations (visual and audible) for conditions where there is high flow. (how are they different than those for incipient cavitation?) Record the pressure for each gage. Sketch your estimate of the pressure vs. position in the plot below. Sketch the appearance of the flow in the diagram below and to the right.



## **B. Flow Visualization Techniques for Flows Around Bodies of Different Shapes**

### 1. Boundary Layer Videos

Answer the following questions, use another sheet of paper if necessary.

1.) Sketch an example from the film of a **laminar** boundary layer.

2.) Sketch an example from the film of a **turbulent** boundary layer.

3.) Name **three** things you learned from the film that can be used to determine whether a boundary layer is laminar or turbulent and how they are used.

### 2. Hydrogen Bubble Device

This flow visualization technique uses an electric current along a thin wire to generate a plane flow of hydrogen and oxygen gas bubbles. These bubbles are tracked to follow the flow pattern.

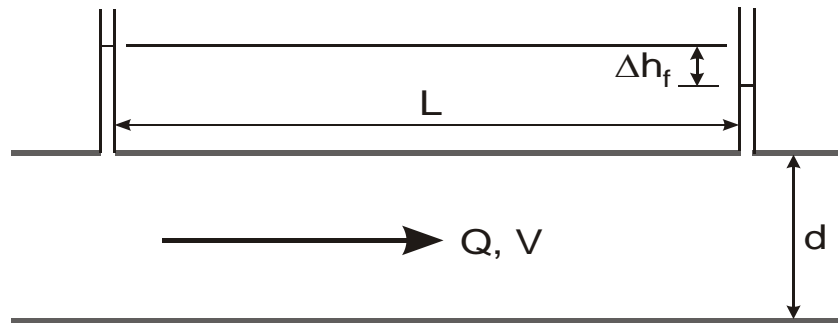
**Look at the flow patterns around the different objects. Observe how the flow separation changes with the different shapes (cylinder, airfoil, rectangular block). Sketch the flow around three different shapes. Describe in words and in details on your sketches how the flow separation changes with the different shapes.**

## CE 3502 FLUID MECHANICS

### Laboratory Experiment #6

#### MEASUREMENTS OF PIPE FRICTION

The friction factor for flow in a given pipe may be determined from a Moody diagram if the Reynolds Number and the pipe roughness are known. In this laboratory session the frictional head loss will be measured and compared to that which would be determined from the Moody diagram (e.g., Section 10.6 in your textbook).



For fully developed flow in a horizontal circular pipe of diameter  $d$  with pressure drop  $\Delta p$  over distance  $L$ , the friction factor is given by the equation:

$$f = \frac{d}{L} \cdot \frac{\Delta p}{\frac{1}{2}\rho V^2}$$

Where  $V$  = mean velocity in the pipe and  $\Delta p = \gamma_w \Delta h_f$ , and  $\gamma_w$  is the specific weight of the manometer fluid (in this case, water). In addition, dimensional analysis indicates that  $f$  is a function of Reynolds Number and relative wall roughness:

$$f = F(\text{Re}, k_s/d)$$

Where  $\text{Re} = V^* d/\nu$  and  $k_s$  = wall roughness

These relationships will be tested and analyzed in the following experiments in which you will measure the friction coefficient for air, water, and oil flow in different pipes. (Convert all values to SI units.)

## AIR FLOW (high Reynolds number flow)

### A. Smooth Pipe (use pipe 3)

#### **Procedure:**

For three fan speeds (60, 80, 100 Volt settings on the rheostat):

1. Measure the flow rate using the Hastings Meter with the appropriate correction factor ( $k = 1.1180$ ). Use this flow rate to calculate the velocity.
2. Measure the  $\Delta h_f$  for two different lengths of pipe. Use a tape measure to measure these lengths ( $L_1$  and  $L_2$ ).

#### **Calculations:**

1. Use  $\Delta h_f$  to calculate  $\Delta p$  over the two different lengths of pipe ( $L_1$  and  $L_2$ ).
2. Calculate the friction coefficient ( $f$ ) and the Reynolds number ( $Re$ ) for each set of measurements (6 total) and plot the points on the Moody Diagram.
3. Determine the equivalent sand roughness  $k_s$  (in mm) for the pipe from these measurements using the Moody Diagram.

### B. Rough Pipe (use pipe 1 or 2)

#### **Procedure:**

For three fan speeds (60, 80, 100 Volt settings on the rheostat):

1. Measure the flow rate using the Venturi meter (use the equation from the side of the manometer bank:  $Q = 2.0786(\Delta h_v)^{0.5}$  where  $\Delta h_v$  is in mm and  $Q$  is in L/s) and use it to calculate the velocity.
2. Measure  $\Delta h_f$  for two different lengths of pipe ( $L_1$  and  $L_2$ ), and measure these lengths with the tape measure.

#### **Calculations:**

1. Use  $\Delta h_f$  to calculate  $\Delta p$  over the two different lengths of pipe ( $L_1$  and  $L_2$ ).
2. Calculate the friction coefficient ( $f$ ) and the Reynolds number ( $Re$ ) for each set of measurements (6 total) and plot the points on the Moody Diagram.
3. Determine the equivalent sand roughness  $k_s$  (in mm) for the pipe from these measurements using the Moody Diagram.

## WATER FLOW

#### **Procedure:**

Using three different valve settings (1, 1.5, and 2 turns):

1. Measure  $Q$ : For the 1-inch diameter pipe, use the differential pressure gage connected to the orifice meter to measure pressure drop  $\Delta p_q$  (in psi) across the orifice plate.
2. Measure the pressure drop  $\Delta p$  over the length of pipe (in psi). Record the length between the two points where the pressure taps are connected to the pipe ( $L = 10$  ft).

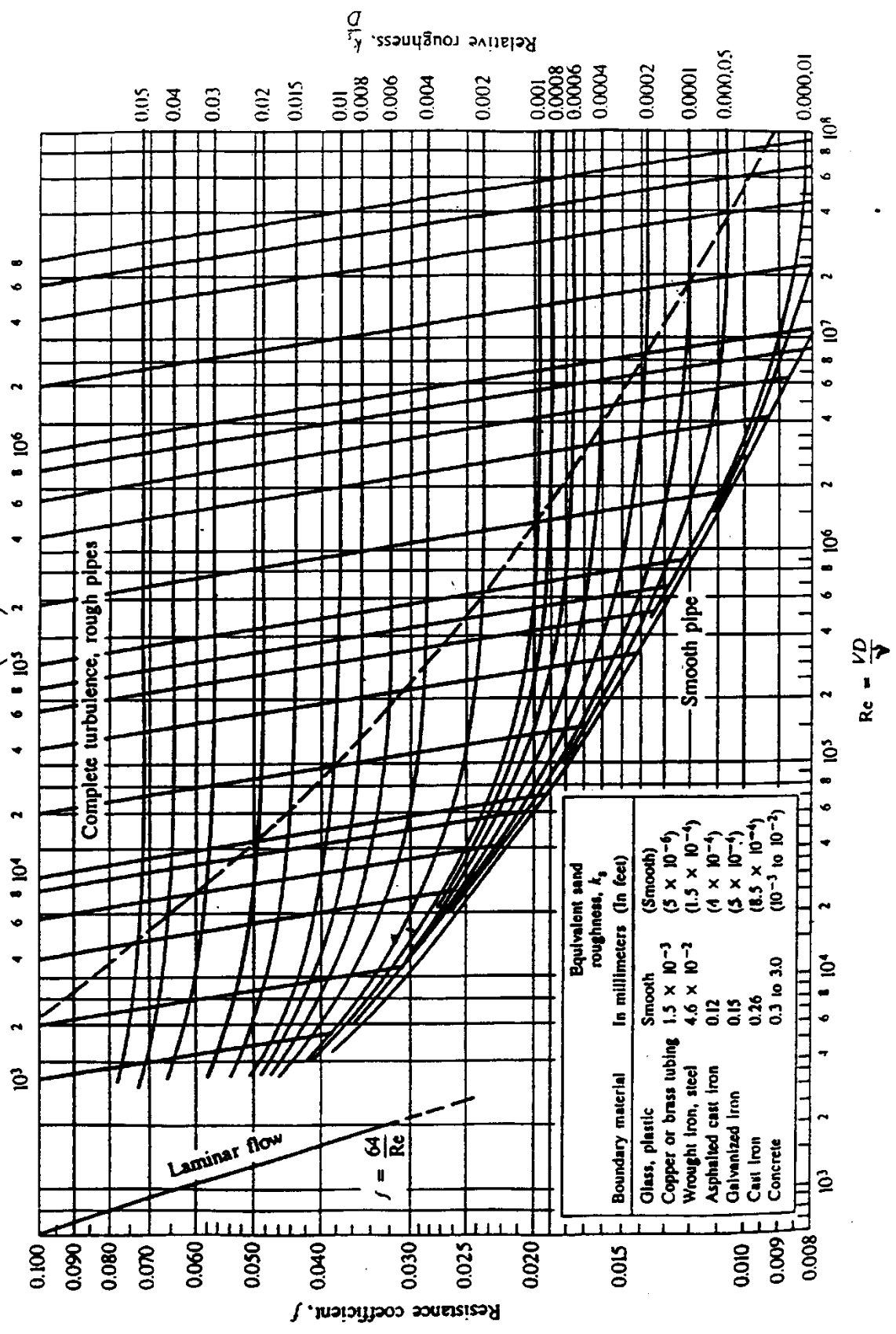
#### **Calculations:**

1. Use  $\Delta p_q$  in the orifice equation to find  $Q$ . Then use the flow rate ( $Q$ ) to calculate the velocity ( $V$ ).  
 $Q = 0.0535 (\Delta p_q)^{1/2}$  (In this formula  $Q$  will be in  $ft^3/s$  when  $\Delta p$  is in  $psi$ )
2. Calculate  $f$  and  $Re$  for each set of measurements (3 total) and plot the points on the Moody Diagram. (A removable copy of the Moody Diagram is included in Appendix D.)
3. Determine the equivalent sand roughness  $k_s$  (in mm) for the pipe.

### THOUGHT QUESTIONS:

1. Based on the results from this lab and from the head loss equations, how would you optimize pumping requirements for long pipelines that are required to discharge large flows?
2. The friction factor for laminar flows, according to the moody Diagram, ranges from 0.025 to 0.1. For extremely turbulent flows, the friction factor is an order of magnitude smaller. Why? Hint: think about what you know about the differences in velocity profile shapes between laminar and turbulent flows – i.e. what effect do the differently shaped boundary layers have of pipe friction factors?

$$Re^5 f^{1/2} = \frac{D^{5/2} \rho \sqrt{2gh_f}}{\nu} \left( \frac{L}{D} \right)^{1/2}$$



## CE FLUID MECHANICS

### Laboratory Experiment #7

#### PIPE BENDS

In this part lab session, you will measure the energy loss coefficients for flow around 90° bends in a copper pipe. There are three pipes. The first pipe is straight, and you will use measurements from it to compute the friction coefficient for the copper pipe, as done in Lab 6, Pipe Friction. The second pipe has one U-shaped loop in it (four 90° bends), and the third pipe has 2 loops in it (eight 90° bends). Using the friction loss measured or calculated for the straight copper pipe, you can also calculate the losses due to the bends from the observed total loss in the pipes with the 90° bends by assuming that the friction factor  $f$  is the same for all three pipes. This is discussed in detail in Section 10.8 of the textbook. You will use the table in Appendix C of the lab manual.

#### A. STRAIGHT PIPE

##### Procedure:

1. Open the valves completely at each end of the pipe. Make sure that the valves for the other pipes are closed.
2. Measure  $\Delta p$  (in psi) between the two gages located at both ends of the pipe.
3. Measure  $\Delta p_q$  (in psi) from the differential pressure gage connected to the orifice meter.  $Q = 0.0535 (\Delta p_q)^{1/2}$  (remember that ***Q will be in ft<sup>3</sup>/s when  $\Delta p$  is psi***)

##### Calculations:

1. Calculate the friction coefficient ( $f$ ) for the pipe ( $d = 0.5$  in,  $L = 10$  ft) using the technique from Lab 6 and the measured head loss ( $\Delta p$ ) in a straight pipe:

$$h_L = \frac{\Delta p}{\gamma_{\text{water}}} = \frac{fL}{d} \frac{V^2}{2g}$$

Note how each of the variables ( $f$ ,  $L$ ,  $d$ ,  $V$ ) on the right side of the above equation affects the head loss. Be careful with unit conversions.

#### B. ONE-LOOP and TWO-LOOP PIPES

##### Procedure:

1. Open both control valves completely for the one-loop pipe (make sure that the valves for all other pipes are closed).
2. Measure  $\Delta p$  and  $\Delta p_q$  as before.
3. Measure the additional pipe length due to the bends in the pipe.
4. Repeat Steps 1 - 3 for the two-loop pipe.

### Calculations:

1. Use the value of  $f$  that you calculated for the straight pipe above and the following new formula for head loss in the pipe:

$$h_L = \frac{\Delta p}{\gamma_{\text{water}}} = \frac{fL}{d} \frac{V^2}{2g} + N_b \cdot K_b \cdot \frac{V^2}{2g}$$

where  $N_b$  = the number of bends.

Calculate  $K_b$ , the loss coefficient for one 90° bend (keep in mind that 1 loop has  $N_b = 4$  bends). Do this calculation for both the one-loop and two-loop pipes.

Should the  $K_b$  values that are calculated for the one-loop and two-loop pipes be similar? Explain.

2. Calculate  $K_b$  taking into account the additional length of pipe added by the bends. How much does the value of  $K_b$  change when you account for the additional length? How important is it to include it?
3. Compare  $fL/D$  to  $N_b \cdot K_b$ . What percentage of the total head loss is due to friction and what percentage is due to the pipe bends?
4. Compare your values to the ones in Appendix C of the Lab Manual. Does your value for  $K_b$  match any of the values listed in the table? Discuss why or why not. What  $r/d$  value do you think is appropriate for the 90° 1/2" copper elbows used in this experiment? Discuss possible reasons for variability from published values.

### THOUGHT QUESTIONS:

For questions 1 and 2, you may need to consult your textbook or find a stormwater engineering manual (may be available online) to determine a reasonable relative roughness for the respective pipe materials.

1. For a long pipeline, explain the importance of determining accurate estimates of the bend coefficients. How important is the accuracy of the bend coefficient for a smooth concrete stormwater drain line with a diameter of 1 meter and a between bend length scale on the order of 100 m? Hint: Estimate the sensitivity of the total flow head loss over a range of bend coefficients and configurations.
2. Answer the same question for a corrugated steel culvert drainage system with a diameter of 30 cm and the same length scale between bends.
3. Summarize the results from 1 and 2. Describe how, as a practicing engineer on a tight budget, you would determine whether or not to conduct a detailed pipe bend analyses in a project. Under what circumstances might you also ignore pipe friction? When is it important to consider both bends and friction?



**CE 3502 FLUID MECHANICS**  
**Laboratory Experiment #8**

**PIPE NETWORKS**

This exercise requires the solution of three exercises involving pipe network problems (e.g., Crowe et al., Chapter 10.10). The solver tool from Excel will be used to solve the systems of equations that arise. The same techniques can be applied to more complicated systems encountered in engineering applications.

You may find that there are multiple possible formulations to solve a single problem. A consistent sign convention is essential. The example provided in EXERCISE 1 uses the following sign conventions.

- Loop: Head loss in the direction of flow is positive (+)
- Head gain in the direction of flow is negative (-)
- Head loss in the opposite direction of flow is negative (-)
- Node: Flow into the node is positive (+)
- Flow out of the node is negative (-)

It is also important to select the right number of appropriate equations (i.e. one equation for each unknown, no more, no less).

**EXERCISE 1**

The solution to this pipe network has been prepared in advance. Be sure to check the equations for each balance and for each loop. (*Hint: There is an error in one of the equations*)

Case 1: B = 20, C = 50, D = 30

Case 2: B = 0, C = 0, D = 100

**EXERCISE 2**

For the diagram labeled ‘Exercise 2’, calculate the flow rates and flow directions in all pipes. Make a diagram that shows the positive flow directions that you used to write your equations. Use the equation given below for the pump. You will have to calculate the f and r-values for each section of pipe using the following information (r-value for pipe 6 = 0).

<b>Exercise 2 Pipe Characteristics</b>					
Pipe	1	2	3	4	5
Length (m)	300	600	500	400	300
Diameter (m)	0.15	0.3	0.6	0.3	0.3

To calculate f, use the value  $k_s = 0.3$  mm (for concrete pipe). You will need to use the Moody Diagram to calculate f. Assume that the flow is fully turbulent, so you can calculate  $k_s/d$  and follow it straight across to find f. Also use  $Q_{out} = 1$  m<sup>3</sup>/s.

$$H_{pump} \text{ (m)} = 150 - 2Q - 0.5Q^2$$

$$r = \frac{8fL}{D^5 \pi^2 g}$$

To evaluate whether or not this pump is a good choice for this situation, check its efficiency using the following table. Linearly interpolate efficiencies that fall between values given in the table.  $Q_{\max}$  is the flow rate at which the head is zero. It is calculated by solving the pump equation for  $Q$  with the head set equal to zero.

Pump Efficiency					
Percent of $Q_{\max}$	0	25	50	75	100
$\eta$	0	38	75	85	0

Draw a diagram of the exercise with all the flows labeled along with their directions. Explain whether or not the pump is a good choice for the situation based on its efficiency.

Now, the situation is modified a little. Instead of using the pump equation given above, you will be solving for the head of the pump and the flow through the pump independently. This adds an unknown and therefore requires an additional loop or node equation. Suppose you wanted to replace pipe 1 in the network. In order to do that, you could control the flow rates with a pump so that the flow in pipe 1 is zero. Set the flow in the pipe 1 equal to zero and solve for the pump head and flow required in that situation. This will determine the size of pump needed to stop the flows. Now repeat this analysis, but this time replacing pipe 3 instead of pipe 1. Is the pump provided suitable for each of the two situations?

### EXERCISE 3

Evaluate the flows for the pump shown in the figure labeled ‘Exercise 3’. Write two loop equations and one balance equation for the system. Discuss whether or not the two pumps given below would be good selections. Do they run at a high efficiency? You can use the efficiency information given above. Give your final values of  $H_{\text{pump}}$  for each pump. Sketch a diagram of the problem (including  $Q$  values and flow directions).

Pump Equations:

$$H_{\text{pump1}} = 100 - 500Q$$

$$H_{\text{pump2}} = 160 - 750Q$$

Required Constants:

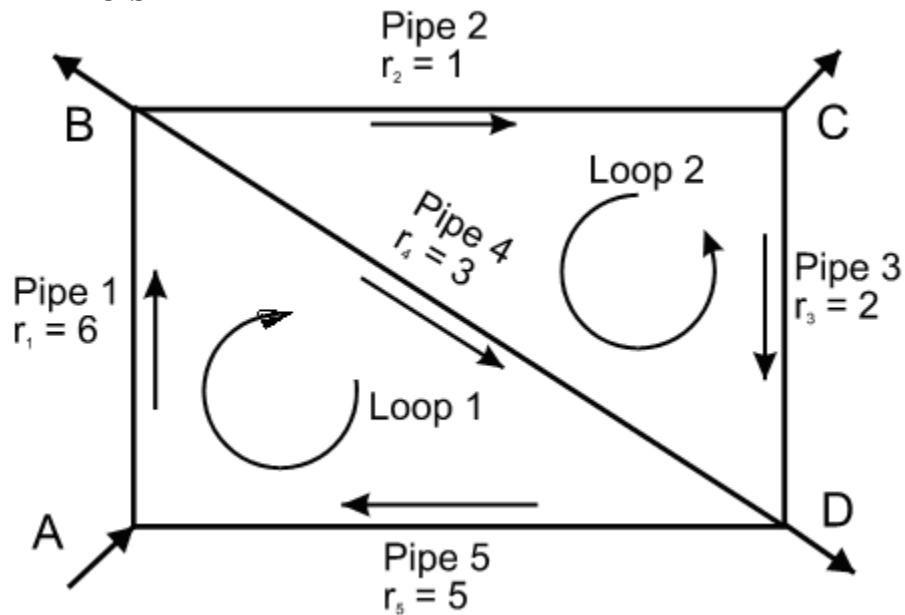
$$L_1 = 1500 \text{ m}$$

$$L_2 = L_3 = 600 \text{ m}$$

$$D_1 = D_2 = D_3 = 0.2 \text{ m}$$

$$f = 0.026$$

### EXERCISE 1



### EQUATIONS:

**Loop 1:**  $r_1 Q_1^2 + r_4 Q_4^2 + r_5 Q_5^2 = 0$  ... which should be written as:

$$R_1 * Q_1 * \text{ABS}(Q_1) + R_4 * Q_4 * \text{ABS}(Q_4) + R_5 * Q_5 * \text{ABS}(Q_5) = 0$$

**Loop 2:**  $-R_2 * Q_2 * \text{ABS}(Q_2) - R_3 * Q_3 * \text{ABS}(Q_3) + R_4 * Q_4 * \text{ABS}(Q_4) = 0$

**Balance Node A:**  $Q_A + Q_5 - Q_1 = 0$

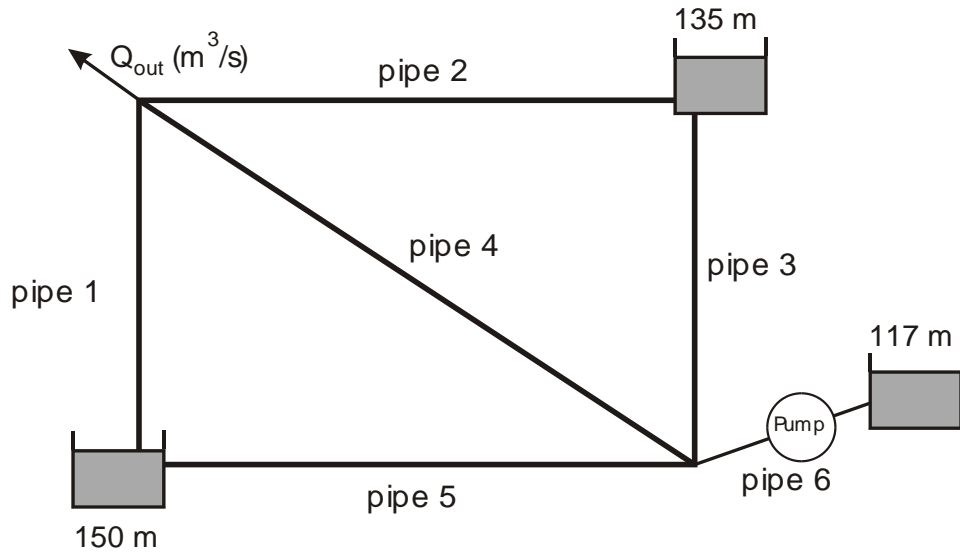
**Balance Node B:**  $Q_1 - Q_2 - Q_B - Q_4 = 0$

**Balance Node C:**  $Q_2 - Q_3 - Q_C = 0$

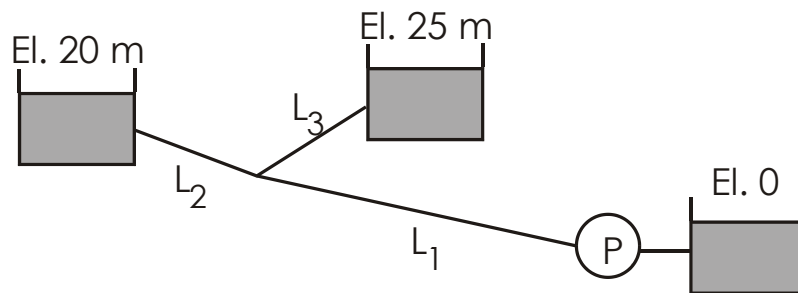
**Balance Node D:**  $Q_3 + Q_4 + Q_5 - Q_D = 0$

**Overall Balance:**  $Q_A - Q_B - Q_C - Q_D = 0$

## EXERCISE 2



## EXERCISE 3



## Pipe Networks in Excel

### Procedure:

1. Write balance equations for each pipe junction (into the junction is positive and out is negative).
2. Find a reasonable set of Q's to satisfy the balance equations.
3. Write down the head equations around each loop using a term  $R1*Q1*ABS(Q1)$  for each pipe.
4. Assign separate cells in the spreadsheet to each Q value. Label the cells Q1, Q2, etc.
5. Assign separate cells in the spreadsheet to each r-value. Label them R1, R2, etc.
6. Set up equations (HEAD and FLOW), each in a separate cell of the spreadsheet. Write the equations in such a way that if the correct values were chosen, the corresponding cell value would be zero.
7. Go to the tools menu.
8. Select solver (if it's not installed, you can sometimes install it using Add-Ins under the Tools menu).
9. Select the first HEAD equation as the target cell.
10. Set this equal to zero.
11. Select the Q cells as the changing cells.
12. Select all of the other equation cells as constraints (set them equal to zero).
13. Solve.

### THOUGHT QUESTIONS:

1. The solver algorithm uses a technique similar to Newton's method to minimize residuals (errors) in the solution of the system of equations. As a consequence, several distinct solutions may yield adequately small mean residuals (i.e. non-unique solutions). Explain how you would determine if the solution is unique. If it's not unique, how would you determine whether your solution is the correct one? What steps might you take in setting up a more complicated system to ensure unique solutions exist? Hint: think back to matrix theory; what criteria ensure uniqueness of solution? Newton's method relies on finding local slopes to determine if the solution has reached a minimum in the error function. Consequently, it can get "fooled" by local minima if the error function is complicated. Can you suggest a more robust technique for minimizing errors – i.e. a technique that will not be "fooled" by local minima?
2. Suggest other applications in engineering to which such an analysis as this is well-suited.

**CE 3502 FLUID MECHANICS**  
**Laboratory Experiment #9**

**PUMP PERFORMANCE & PUMP SIZING**

In this laboratory you will perform two separate exercises. In one exercise, you will respond to a Request for Design letter enclosed as the last section of these instructions. In the other exercise, you will perform a laboratory experiment to compute the efficiency of a pump at different flow rates, and generate an efficiency curve for the pump. (See Chapter 14 in Crowe et al. for some relevant information on pumps and Chapter 10 for some information on pumps in parallel and series.)

Information on the pump stands can be found in the appendix. To perform the computations for each pump stand, you will need to take the following 6 measurements for 10 different flow rates:

1. Voltage. This is read directly from a voltmeter.
2. Current (amps). This is read directly from an ammeter.
3. Rotational Speed (RPM). Since this will vary only slightly, you may use an average value in your calculations.
4. Force (lbf) provided by the moment arm of the motor. This is read from a force gauge.
5. Pressure across the pump (psi). Take the reading from the gauge connected to the flexible tubing attached to pump intake and outlet.
6. Pressure across the orifice meter (psi). This is used to find the flow rate. Take the orifice pressure gauge reading from the gauge connected to the orifice meter. Use the orifice meter equations below to compute flow rates:

Large pump stand:	$Q = 15.6 * \Delta p_q^{0.43}$
Medium pump stand:	$Q = 12.5 * \Delta p_q^{0.42}$
Small pump stand:	$Q = 8.0 * \Delta p_q^{0.415}$

where  $Q$  is the volumetric flow rate in gpm  
and  $\Delta p_q$  is the pressure differential (psi) across the orifice plate.

Be sure that all measurements have units appropriate for each formula above.

**PUMP TEST PROCEDURE:**

1. First turn on the strain gage and zero it.
2. Turn on the pump (silver switch).
3. Turn up the pump speed to the highest RPM setting that it will reach.
4. Take all of the measurements at that RPM setting. The RPM readings may fluctuate slightly as you change the flow rate, but use a constant “average” value in your calculations. You will be taking 6 readings (volts, amps, force, RPM,  $\Delta p$  across pump, and  $\Delta p_q$  across the orifice meter) for 11 different flow rates. Make sure to record correct units!
5. Adjust the flow rate by looking at the pressure gage for the orifice meter (the orifice meter is located near the top of the pipe, so follow the tubes down to see which gage to read). Your

first reading should be at the highest flow rate, and the final (11<sup>th</sup>) reading should be a value of about 0.5 psi. In order to get the 11 flow rates, you have to evenly distribute the other 10 readings over this range (you can use  $[\max \Delta p_q - 0.5]/10$  to get the increment value). These readings don't have to be exact, but in order to get a nice graph they should be spread evenly over that range. You will use these  $\Delta p_q$  readings from the orifice meter to get Q values from the formulas.

6. Take the six readings for all 11 flow rates.
7. Repeat the procedure for the other 2 pump stands. You will have 3 sets of results, one for each pump.

### CALCULATIONS:

1. Calculate the Fluid power ( in horsepower = hp)

$$P_{fl} = \frac{\Delta p}{0.4335} \cdot \frac{Q \cdot 8.33}{33000}$$

$P_{fl}$  = fluid power (hp)

Q = volumetric flow rate (gpm)

$\Delta p$  = pressure differential across the pump (psi)

0.4335 is a constant to convert psi to ft. of water

2. Calculate the Mechanical power (in hp)

$$P_{mech} = \frac{F \cdot L \cdot \omega}{5250}$$

$P_{mech}$  = mechanical power (hp)

L (length of moment arm) = 0.44 ft. (large pump) or 0.67 ft. (small and medium pump)

$\omega$  = angular velocity (rpm)

F = force (lb)

5250 is a conversion factor

3. Calculate the Electric power:

$P_{elec}$  (in watts) = volts \* amps

(convert watts to hp for comparing to  $P_{fl}$  and  $P_{mech}$ ; 1 watt =  $1.341 \cdot 10^{-3}$  hp)

4. Calculate the mechanical and electrical efficiency for all points and graph the efficiency curves ( $\eta$  vs. Q).

$$\eta_{mech} = \frac{P_{fl}}{P_{mech}} \quad \eta_{elec} = \frac{P_{fl}}{P_{elec}}$$

5. Which curve should be higher? Why?

6. Calculate the cost of running these pumps at maximum efficiency for one year.

Cost of electricity = \$0.0635 per kwh (kilowatt-hour)

*Northern Construction Company  
18 Duck Street  
Wild Goose, Minnesota 56708*

April xx, 20xx

Handi Mann, Design Engineer  
Water Resource Specialists  
130 Main Street  
Minneapolis, MN 55414

Dear Dr. Mann,

We have an opportunity to purchase two pumps at a very reasonable price from a new company. This firm utilizes a universal design to manufacture pumps with different capacities. The pump sizes include 11.0", 10.2", 9.6" and 8.1" diameter pumps. All four pumps operate at 1750 rpm.

In the near future, we will be working on two different excavation projects that have significantly different pumping requirements. These are:

1. 200' of head at 3600 gpm
2. 80' of head at 16500 gpm

Please select a maximum of two pumps that will satisfy both of these requirements, since we wish to use them on both projects. The non-dimensional pump data, as supplied by the manufacturer, are enclosed.

Please call me at 218-623-3245 or E-mail me at [ishovel@northern.com](mailto:ishovel@northern.com) if you require any further information.

Very truly yours

Ican Shovel  
President



**EULER PUMPS, INC**  
**3502 Turbine Street**  
**Minneapolis, Minnesota**

<b>Maxflow Pump Data</b>		
$\Psi$	$\phi$	$\eta$
0.180	0.0	0.0
0.179	0.01	
0.178	0.02	
0.176	0.03	
0.174	0.04	
0.173	0.05	0.45
0.170	0.06	
0.168	0.07	
0.166	0.08	0.65
0.163	0.09	
0.160	0.10	0.74
0.154	0.12	
0.146	0.14	0.84
0.138	0.16	0.86
0.130	0.18	0.84
0.120	0.20	0.83
0.093	0.25	0.78
0.060	0.30	
0.023	0.35	

where

$$\Psi \equiv \frac{gH}{\omega^2 D^2}$$

$$\phi \equiv \frac{Q}{\omega D^3}$$

Notes: - See Ch. 10 of your text for a discussion of pumps in parallel and in series.  
 - The units of RPM are neither S.I. nor Eng units; convert RPM to rad/s to make it dimensionless (i.e. multiply by  $2\pi/60$ )

**THOUGHT QUESTIONS:**

1. Explain why pump efficiency cannot be 100%. Why is efficiency 0% when  $Q = Q_{\max}$ ? Explain the relationship, qualitatively, between flow and head. Why are flow and head inversely related?

## **Appendix A: Useful Equations**

**Flow Balance (continuity equation or conservation of mass):** For incompressible flows that do not have loss, production, or storage of material (e.g. within a pipe).

$$Q_1 = Q_2$$

Or,

$$A_1 U_1 = A_2 U_2 \quad U_1 = \frac{A_2}{A_1} U_2$$

The latter expression can often be used in conjunction with the Bernoulli equation to develop a pressure-velocity relationship in terms of only one velocity variable. This is useful in many situations, particularly in labs 2 and 3.

**Bernoulli Equation:** For two points along a streamline in steady, incompressible, irrotational flow (Note: we often assume these conditions to simplify reality).

$$\frac{P_1}{\gamma} + \frac{U_1^2}{2g} + z_1 = \frac{P_2}{\gamma} + \frac{U_2^2}{2g} + z_2 = C$$

Or,

$$P_1 + \frac{\rho U_1^2}{2} + \rho g z_1 = P_2 + \frac{\rho U_2^2}{2} + \rho g z_2$$

**Conservation of Momentum Equation (a.k.a. Newton's Second Law):**

$$\sum F_i = ma$$

Or, in a form that is more useful for fluid dynamics, for forces only in one direction, acting on a control volume (CV) and the Control Surface (CS) that bounds the CV,

$$\sum F_i = \frac{\partial}{\partial t} \int_{CV} \rho \vec{U} dV + \int_{CS} \rho \vec{U} (\vec{U} \cdot d\vec{A}) = \frac{\partial}{\partial t} \int_{CV} \rho \vec{U} dV + \int_{CS} \rho \vec{U} (\vec{U} \cdot \hat{n}) dS$$

To simplify this equation, two assumptions must be made. First, assuming steady state conditions will allow time derivatives to vanish. Second, the CV is assumed to be of differential size, which is sufficiently small that variations within the CV and along the CS are considered negligible. The following results:

$$\sum F_i = \int_{in}^{out} \rho \bar{U} (\bar{U} \cdot \hat{n}) \cdot dS$$

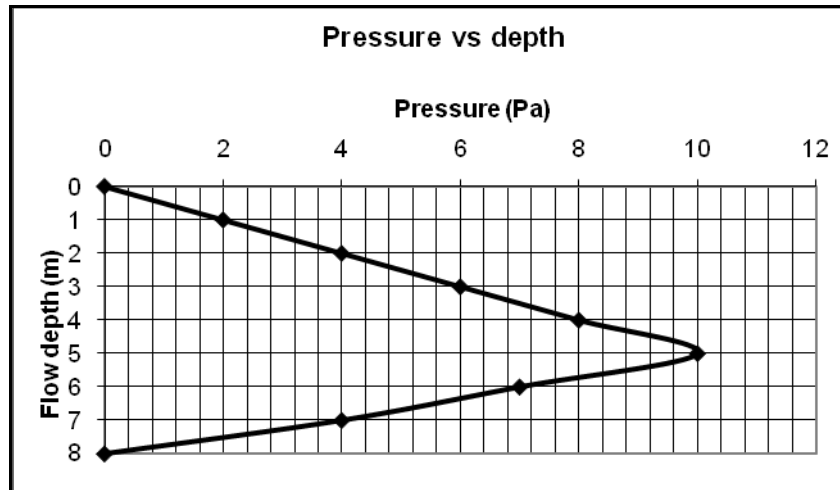
This result can be manipulated to give the expression used in Lab 4. Hint: Recall that  $(\bar{U} \cdot \hat{n}) \cdot S = (\bar{U} \cdot \hat{n}) \cdot A = Q$ .

## Appendix B: Useful techniques for working with lab data.

Numerical Integration:

Engineers and scientists are often required to find areas under curves. For example, force on the sluice gate can be obtained from a pressure distribution along its surface. Recall,  $\int dF = \int PdA = b \int Pdy$ . However, lab data is typically a series of discrete points rather than a smooth function that is easily integrated using techniques of calculus.

P	y
0	0
2	1
4	2
6	3
8	4
10	5
7	6
4	7
0	8



Many students, with the aid of a spreadsheet function, will be tempted to fit a high order polynomial function to the data. Integrating this function will produce the area under the curve. This technique is not generally advisable for a variety of reasons.

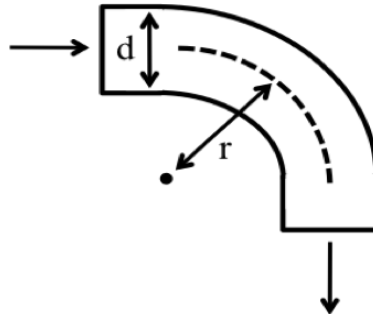
A better solution is a graphical approach. Here the data is plotted as shown above. The software will draw gridlines that form boxes. The area of each box is found (Remember to include units!!!  $A_{\text{box}} = 1 \text{ m} \times 0.4 \text{ Pa} = 0.4 \text{ Pa} \cdot \text{m}$ ) and multiplied by the total number of boxes ( $\sim 100$ ) that lie within the area of interest. If better accuracy is needed, use more closely spaced gridlines. Too many boxes = minimal gains in accuracy = Too much work.

Spreadsheets can also be used to efficiently compute the area using a rectangle or trapezoid approximation. This method eliminates tedious box counting and can be very accurate.

## Appendix C: Minor Loss Coefficients

**Table C.1.** Minor Loss Coefficients for 90° Smooth Bend.\*

$r/d$	$K_b$
1	0.35
2	0.19
4	0.16
6	0.21
8	0.28
10	0.32



\* from Crowe CT, Elger DF, Williams BC, Roberson JA. Engineering Fluid Mechanics, 9<sup>th</sup> Ed. John Wiley and Sons, New York, 2009.

## **Appendix D: User's Manual for Pump Testing Stand**

### Overview

The Pump Testing Stand was designed and constructed to demonstrate operational characteristics of centrifugal pump systems. The Pump Testing Stand achieves these goals by allowing the students to vary the motor RPM and pressure across the pump by adjusting a throttling valve to change the loss coefficients of the piping system which the pump supplies with water. The Pump Testing Stand has sensors showing the motor voltage, current, RPM, torque, pump pressure, and flow in the system. Knowledge of these parameters allows the students to calculate electrical power into the system, mechanical horsepower to the pump, and useful power supplied by the pump. There are three different Pump Testing Stands with different impeller diameters allowing the instructor to highlight the effects of changing impeller diameters upon the flow and pressure characteristics of a pump.

### Pump Testing Stands

Each of the stands has a 3/4 Horsepower (hp) variable speed DC electric motor. The motor has a maximum rotational speed of 1750 revolutions per minute (RPM). The motor controllers allow the rotational speed of the motor to be infinitely varied from 350 to 1750 RPM. Once set, the rotational speed stays constant regardless of load. Motor manufacturer's specifications are listed separately.

The motors of each pump testing stand are directly coupled to a centrifugal pump. Each of the three stands has a different diameter impeller on the centrifugal pump. The fluid flow rate is measured by an orifice plate. Discharge from the pumps is controlled by gate valves.

### Mechanical Power

The motor shaft is suspended by two ball bearings, leaving the housing of the motor free to rotate. This allows the measurement of the reaction torque from the operation of the motor. This is accomplished with a load cell to measure force, calibrated in pounds. The load cell is mounted 8" from the center of the motor shaft, with the load cell measuring force perpendicular to the radius, as shown in Figure 1.

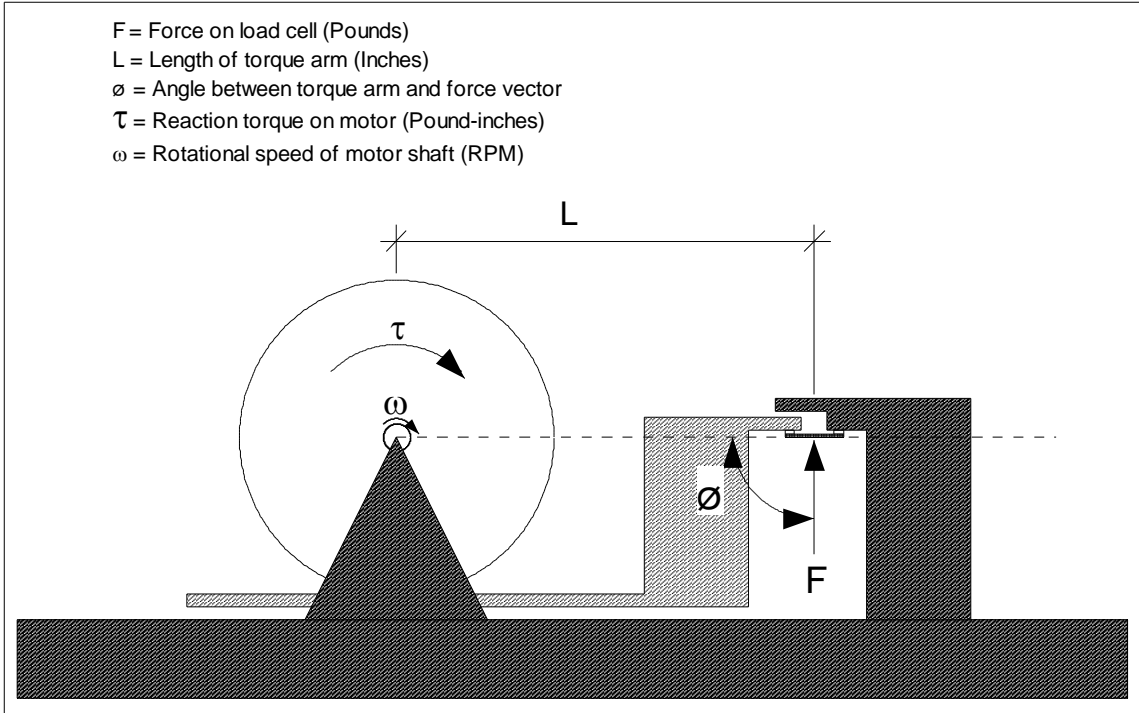


Figure 1: Torque measurement schematic

The power, measured in horsepower, delivered to the pump from the motor can then be found using Equation 1, below.

$$\text{Horsepower} = \frac{\text{Torque (lb - in)} * \omega \text{ (RPM)}}{63025} = \frac{L \text{ (in)} * F \text{ (lbs)} * \omega \text{ (RPM)} * \sin(\varnothing)}{63025} \quad \text{Equation 1.}$$

Where  $L$  = Torque arm length (inches)

$F$  = Force on load cell (pounds)

$\omega$  = Rotational speed in revolutions per minute (RPM)

$\varnothing$  = Angle between torque arm and force vector

Electrical Power

The electrical motor operates on direct current (DC) with a controller to regulate rotational speed. The controller varies the CD voltage supplied to the motor, which controls the rotational speed of the motor. The current (amperage) is determined by the load required of the motor. The electrical power is given in watts and in general is found by Equation 2.

$$\text{Watts} = V * I \quad \text{Equation 2.}$$

where Watts = power into the electric motor

$V$  = voltage supplied to motor

$I$  = amperage (under load) of motor

The motor used on this pump stand is a compound wound motor with adjustable speed. The voltage and current supplied to the field windings are constant at 103 volts and 0.5 amps. The meters on the pump test stand measure the voltage and current supplied to the motor armature, both of which vary with motor speed. The electrical power is given in watts, and for this motor in particular is found by Equation 3.

$$\text{Watts} = V * I + (103 \text{ volts}) * (0.5 \text{ amps}) \quad \text{Equation 3.}$$

where Watts = power supplied to the electric motor

V = voltage supplied to motor armature

I = amperage draw of motor armature

#### Fluid Power

The fluid power is the power performed by the pump on the fluid. The pump's impeller is rotated by the electrical motor, causing a pressure increase from the pump's inlet to its outlet. This pressure differential is the force that moves the fluid. The power is a function of the differential pressure across the pump and the fluid flow rate. The power can be found using Equation 4, below.

$$\text{Fluid Power} = \Delta\text{Pressure} * \frac{2.307 \text{ ft. of water}}{1 \text{ pound per sq. inch}} * \text{Flow Rate} * \rho \quad \text{Equation 4.}$$

where Fluid Power = horsepower exerted on the fluid by the pump (hp)

$\Delta$ pressure = the pressure differential across the pump in pounds per square inch (psi)

Flow Rate = volumetric fluid flow rate in gallons per minute (gpm)

$\rho$  = the density of water at 70°F = 8.33 pounds per gallon

The fluid flow rate is measured by using an orifice plate and measuring the pressure differential across it. The actual orifice equation is very complex with several terms and coefficients depending on the pipe's interior diameter and the diameter of the orifice plate. To simplify the flow rate calculation, we will use a power equation shown in Equation 5, below.

$$\text{Flow Rate} = M * \Delta H^b \quad \text{Equation 5.}$$

where Flow Rate = volumetric fluid flow rate in gallons per minute (gpm)

M = multiplication coefficient

$\Delta H$  = the pressure differential across the orifice plate in pounds per square inch (psi)

b = power coefficient



## Efficiencies

In the above sections, power was calculated using units commonly used in practical applications. For example, small electric motors are rated by their power in horsepower, now watts. Efficiency is defined as the ratio of the useful energy output of a device to the energy input supplied to the device. To calculate efficiencies, the energy units must be the same. For the previous calculations, either watts or horsepower will suffice.

### Power Conversion Factors:

- 1 Watt = 0.00134 Horsepower
  - 1 Watt = 1 Joule per second
  - 1 Horsepower = 745.7 Watts
  - 1 Horsepower = 42.436 BTU per minute
- Equation 6.

After converting the electrical, mechanical and fluid powers to the same units, the efficiencies can be found.

$$\eta (\text{efficiency}) = \frac{\text{Useful energy output}}{\text{Energy input supplied}} \quad \text{Equation 7.}$$

$$\eta_{\text{mechanical}} = \frac{\text{Mechanical energy supplied to the pumpshaft}}{\text{Electrical energy supplied to the motor}}$$

$$\eta_{\text{pump}} = \frac{\text{Fluid energy supplied by the pump to the fluid}}{\text{Mechanical energy supplied to the pumpshaft}}$$

$$\eta_{\text{overall}} = \frac{\text{Fluid energy supplied by the pump to the fluid}}{\text{Electrical energy supplied to the motor}}$$

### Large Pump Testing Stand

This stand has a 3/4 Horsepower (hp) variable speed CD electric motor. The motor has a maximum rotational speed of 1750 revolutions per minute (RPM). The pump is a centrifugal type with an impeller diameter of 4.25” (inches). Manufacturer’s specifications are listed separately.

The orifice pipe is a 1 1/4 PVC schedule 40 with an interior diameter of 1.38”. The orifice plate is concentric, made of 1/8” brass sheeting with a 1/16”, 45° bevel on the downstream face. The interior diameter of the orifice is 0.82”. The approximate flow rate is given by Equation 8.

$$\text{Flow Rate} = 15.6 * \Delta H^{0.43} \quad \text{Equation 8.}$$

where Flow Rate = volumetric fluid flow rate in gallons per minute (gpm)  
for the large test stand

$\Delta H$  = the pressure differential across the orifice plate in pounds per square inch (psi)

### Medium Pump Testing Stand

This stand has a 3/4 Horsepower (hp) variable speed DC electric motor. The motor has a maximum rotational speed of 1750 revolutions per minute (RPM). The pump is a centrifugal type with an impeller diameter of 3.75" (inches). Manufacturer's specifications are listed separately.

The orifice pipe is a 1 PVC schedule 40 with an interior diameter of 1.05". The orifice plate is concentric, made of 1/8" brass sheeting with a 1/16", 45° bevel on the downstream face. The interior diameter of the orifice is 0.70". The approximate flow rate is given by Equation 9.

$$\text{Flow Rate} = 12.5 * \Delta H^{0.42} \quad \text{Equation 9.}$$

where Flow Rate = volumetric fluid flow rate in gallons per minute (gpm)  
for the medium pump test stand

$\Delta H$  = the pressure differential across the orifice plate in pounds per square inch (psi)

### Small Pump Testing Stand

The stand has a 3/4 horsepower (hp) variable speed DC electric motor. The motor has a maximum rotational speed of 1750 revolutions per minute (RPM). The pump is a centrifugal type with an impeller diameter of 3.56" (inches). Manufacturer's specifications are listed separately.

The orifice pipe is a 3/4 PVC schedule 40 with an interior diameter of 0.82". The orifice plate is concentric, made of 1/8" brass sheeting with a 1/16", 45° bevel on the downstream face. The interior diameter of the orifice is 0.55". The approximate flow rate is given by Equation 10.

$$\text{Flow Rate} = 8.0 * \Delta H^{0.415} \quad \text{Equation 10.}$$

where Flow Rate = volumetric fluid flow rate in gallons per minute (gpm)  
for the small pump test stand

$\Delta H$  = the pressure differential across the orifice plate in pounds per square inch (psi)



# Appendix E: Moody Diagram

